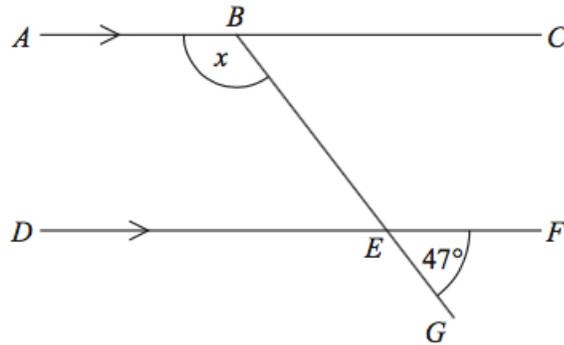# Dr Oliver Mathematics
GCSE Mathematics
2012 June Paper 2H: Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. ABC and DEF are parallel lines. (3)

BEG is a straight line.
Angle $GEF = 47°$.
Work out the size of the angle marked x.
Give reasons for your answer.

Solution

Angle $BED = 47°$ (alternate angles).
Angle $ABE = 180 - 47 = \underline{133°}$ (interior angles).

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2. (a) Use your calculator to work out (2)

$$\frac{38.5 \times 14.2}{18.4 - 5.9}.$$

Write down all the figures on your calculator display.
You must give your answer as a decimal.

Solution

$$\frac{38.5 \times 14.2}{18.4 - 5.9} = \frac{546.7}{12.5}$$
$$= \underline{43.736}.$$

(b) Write your answer to part (a) correct to 1 significant figure. (1)

Solution

$\underline{\underline{40 \ (1 \ \text{sf})}}$.

3. Pradeep wants to find out how much time people spend playing sport.
He uses this question on a questionnaire.

> How much time do you spend playing sport?
>
> ☐   ☐   ☐
>
> 0 – 1 hours   1 – 2 hours   3 – 4 hours

(a) Write down **two** things wrong with this question. (2)

Solution

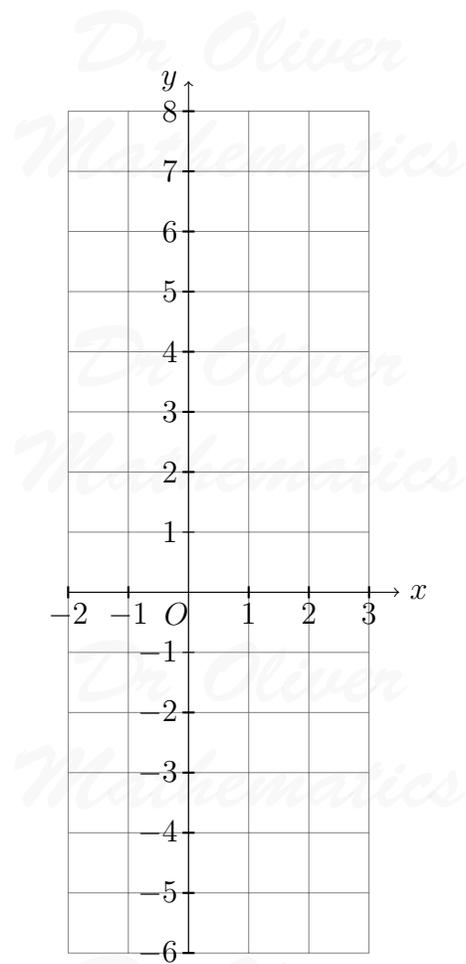E.g., there is no time fame, there is an overlap between 1 hour, $2\frac{1}{2}$ hours?, over 4 hours?

(b) Design a better question for Pradeep's questionnaire to find out how much time people spend playing sport. (2)

Solution

A suitable question with a time frame, e.g., "Did you play sport today/last week/last month? Tick the appropriate box."
At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, Did not play, $0 < x < 1$ hour, $1 < x < 2$ hours, $x \geqslant 2$ hours.

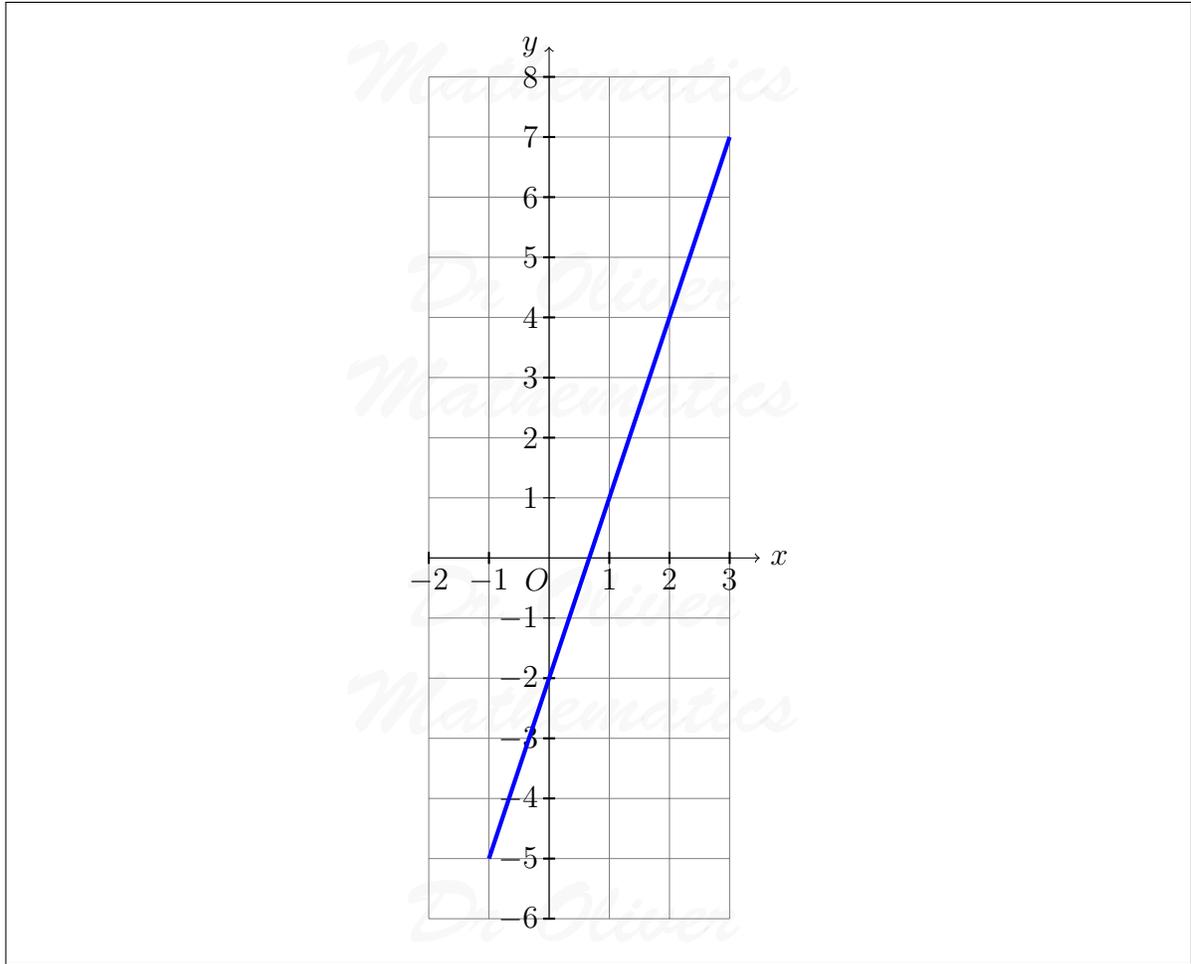4. On the grid, draw the graph of $y = 3x - 2$ for values of x from -1 to 3. (3)

Solution

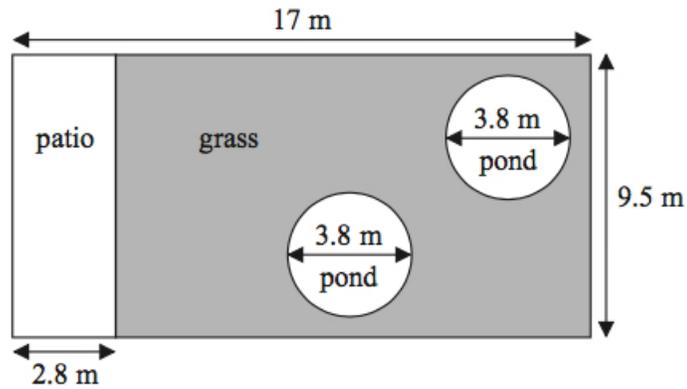| x	-1	0	1	2	3
y	-5	-2	1	4	7

5. Mr Weaver's garden is in the shape of a rectangle.

(5)

In the garden there is a patio in the shape of a rectangle and two ponds in the shape of circles with diameter 3.8 m.

The rest of the garden is grass.

Mr Weaver is going to spread fertiliser over all the grass.
One box of fertiliser will cover 25 m² of grass.
How many boxes of fertiliser does Mr Weaver need?
You must show your working.

Solution

$$\text{Area of the garden} = (17 - 2.8) \times 9.5 - 2 \times \pi \times 1.9^2$$
$$= 112.217\,701 \text{ (FCD).}$$

Finally,
$$\frac{112.217\ldots}{25} = 4.488\,708\,042 \text{ (FCD)}$$
so Mr Weaver needs 5 bags of fertiliser.

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6. Potatoes cost £9 for a 12.5 kg bag at a farm shop. (4)
   The same type of potatoes cost £1.83 for a 2.5 kg bag at a supermarket.
   Where are the potatoes the better value, at the farm shop or at the supermarket?
   You must show your working.

Solution
$$\frac{12.5}{2.5} \times 1.83 = 9.15$$
so the customer should use the farm shop.

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7. The scatter graph shows some information about 8 cars.
   For each car it shows the engine size, in litres, and the distance, in kilometres, the car travels on one litre of petrol.

Distance (kilometres)
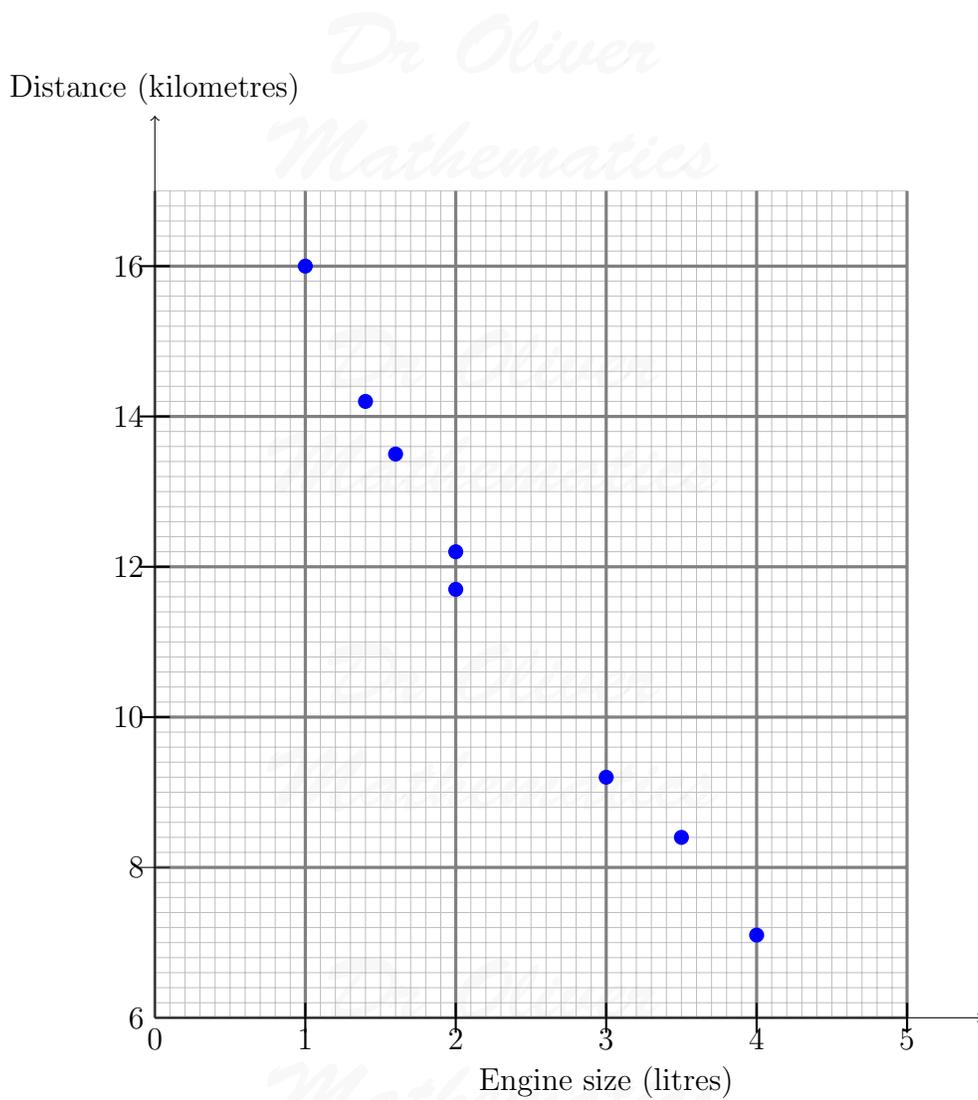
Engine size (litres)

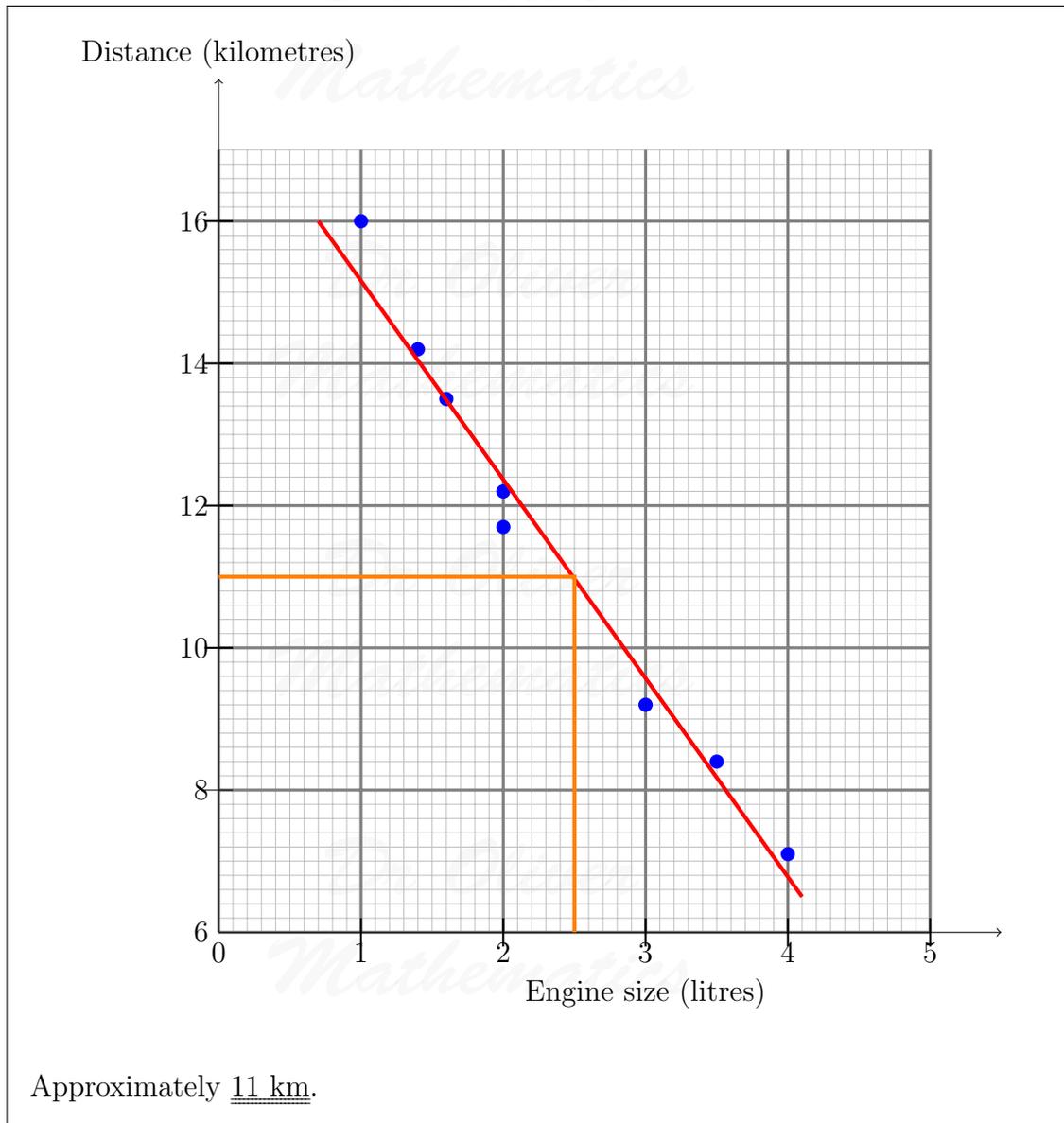(a) What type of correlation does the scatter graph show? (1)

Solution
Negative correlation.

A different car of the same type has an engine size of 2.5 litres.

(b) Estimate the distance travelled on one litre of petrol by this car. (2)

Solution
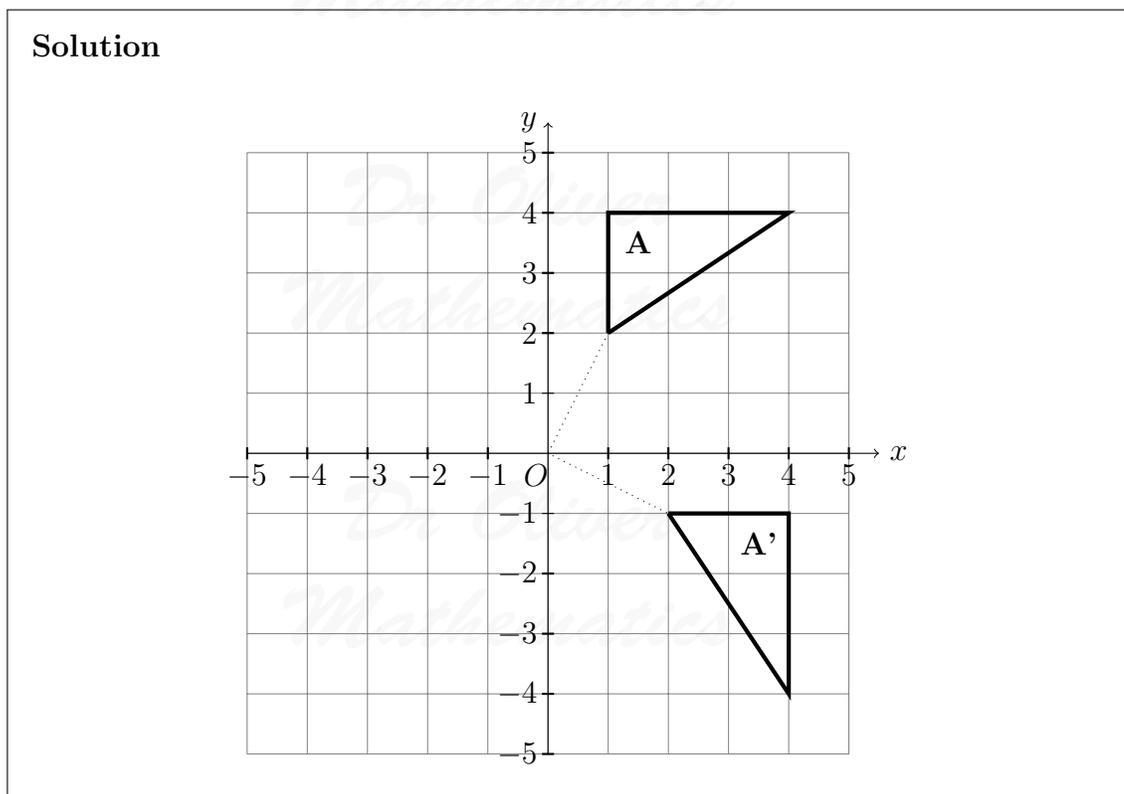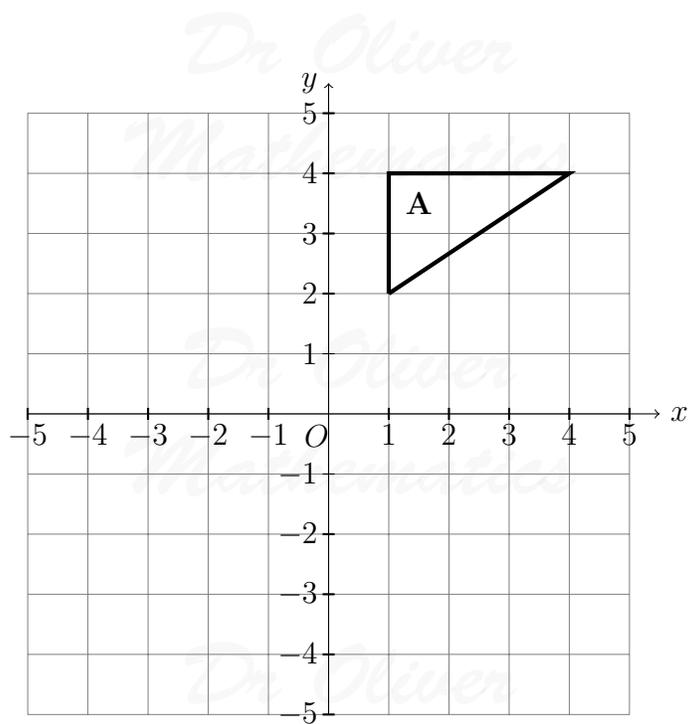Draw a line of best fit and read-off:

Approximately <u>11 km</u>.

8. (a) Rotate triangle **A** $90°$ clockwise, centre O. (2)

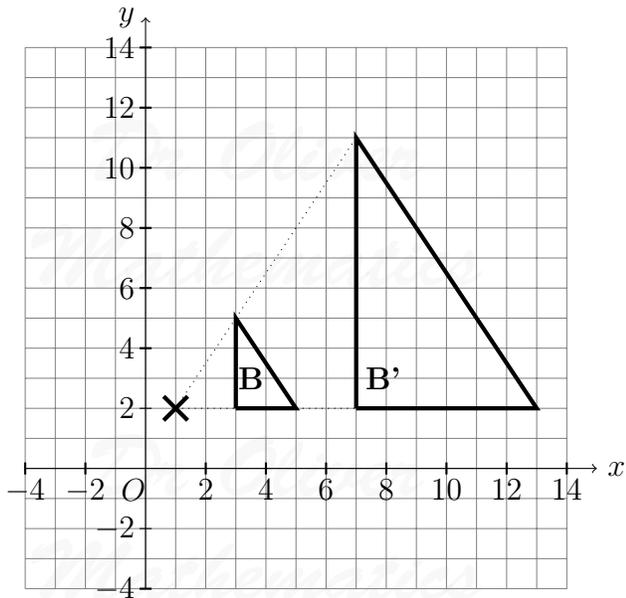**Solution**

(b) Enlarge triangle **B** by scale factor 3, centre $(1, 2)$. (3)
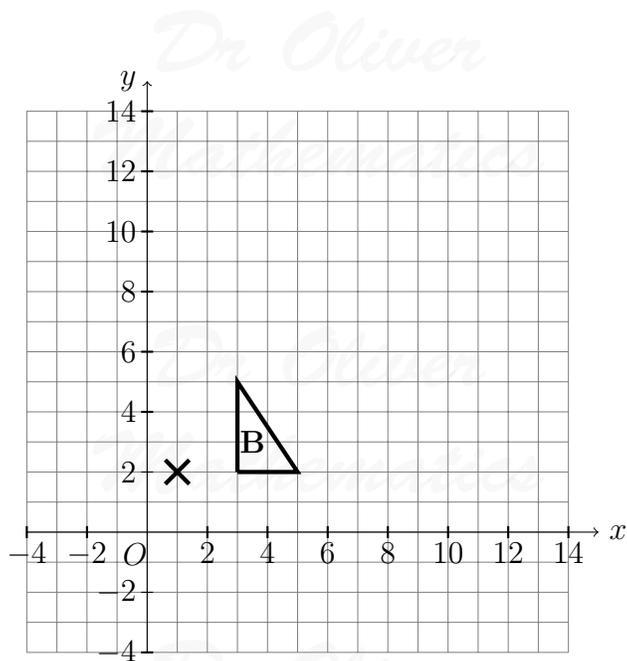
Solution

9. Linda is going on holiday to the Czech Republic. (3)
   She needs to change some money into koruna.
   She can only change her money into 100 koruna notes.
   Linda only wants to change up to £200 into koruna.
   She wants as many 100 koruna notes as possible.

The exchange rate is £1=25.82 koruna.
How many 100 koruna notes should she get?

Solution
$$\frac{200 \times 25.82}{100} = 51.64$$
so Linda should get <u>51</u> 100 koruna notes.

10. m is an integer such that $-2 < m \leqslant 3$.

(a) Write down all the possible values of m. (2)

Solution
$\underline{-1, 0, 1, 2, 3.}$

(b) Solve (2)
$$7x - 9 < 3x + 4.$$

Solution
$$7x - 9 < 3x + 4 \Rightarrow 4x < 13$$
$$\Rightarrow \underline{x < 3\tfrac{1}{4}}.$$

11. The equation (4)
$$x^3 - 6x = 72$$
has a solution between 4 and 5.
Use a trial and improvement method to find this solution.
Give your answer correct to one decimal place.
You must show **all** your working.

Solution

You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.
F(X)= and you type in $X^3 - 6X$; then you press $\boxed{=}$.
Start? and you enter 4; then you press $\boxed{=}$.

End? and you enter 5; then you press $\boxed{=}$.
Step? and enter $0.05 - 1$ decimal place divided by 2; then you press $\boxed{=}$.

x	$f(x)$	Comment
4.6	69.735	too low
4.65	72.644	too high

Clearly,
$$4.6 < x < 4.65$$
and the answer is
$$x = 4.6 \text{ (1 dp)}.$$

12. The probability that a biased dice will land on a five is 0.3.   (2)
    Megan is going to roll the dice 400 times.
    Work out an estimate for the number of times the dice will land on a five.

Solution
$$400 \times 0.3 = \underline{\underline{120}}.$$

13. Bob asked each of 40 friends how many minutes they took to get to work.   (4)
    The table shows some information about his results.

Time taken (m minutes)	Frequency
$0 < m \leqslant 10$	3
$10 < m \leqslant 20$	8
$20 < m \leqslant 30$	11
$30 < m \leqslant 40$	9
$40 < m \leqslant 50$	9

Work out an estimate for the mean time taken.

Solution

Time taken (m minutes)	Frequency	Midpoint	Frequency×Midpoint
$0 < m \leqslant 10$	3	5	$3 \times 5 = 15$
$10 < m \leqslant 20$	8	15	$8 \times 15 = 120$
$20 < m \leqslant 30$	11	25	$11 \times 25 = 275$
$30 < m \leqslant 40$	9	35	$9 \times 35 = 315$
$40 < m \leqslant 50$	9	45	$9 \times 45 = 405$
Total	40		1 130

$$
\begin{aligned}
\text{Mean time taken} &= \frac{\sum fx}{\sum f} \\
&\approx \frac{1\,130}{40} \\
&= 28\tfrac{1}{4} \text{ minutes.}
\end{aligned}
$$

14. (a) Expand and simplify (2)

$$(p + 9)(p - 4).$$

Solution

×	p	$+9$
p	p^2	$+9p$
-4	$-4p$	-36

$$(p + 9)(p - 4) = \underline{\underline{p^2 + 5p - 36}}.$$

(b) Solve (3)

$$\frac{5w - 8}{3} = 4w + 2.$$

Solution

$$\frac{5w - 8}{3} = 4w + 2 \Rightarrow 5w - 8 = 3(4w + 2)$$
$$\Rightarrow 5w - 8 = 12w + 6$$
$$\Rightarrow -14 = 7w$$
$$\Rightarrow \underline{\underline{w = -2}}.$$

(c) Factorise (1)
$$x^2 - 49.$$

Solution

$$\left.\begin{array}{ll} \text{add to:} & 0 \\ \text{multiply to:} & -49 \end{array}\right\} + 7, -7$$
$$x^2 - 49 = \underline{\underline{(x + 7)(x - 7)}}.$$

(d) Simplify (2)
$$(9x^8 y^3)^{\frac{1}{2}}.$$

Solution

$$(9x^8 y^3)^{\frac{1}{2}} = \underline{\underline{3x^4 y^{\frac{3}{2}}}}.$$

15. Henry is thinking about having a water meter. (5)
These are the two ways he can pay for the water he uses.

Water Meter
A charge of £28.20 per year
plus
91.22p for every cubic metre of water used
1 cubic metre = 1000 litres

No Water Meter
A charge of £107 per year

13

Henry uses an average of 180 litres of water each day.
Henry wants to pay as little as possible for the water he uses.
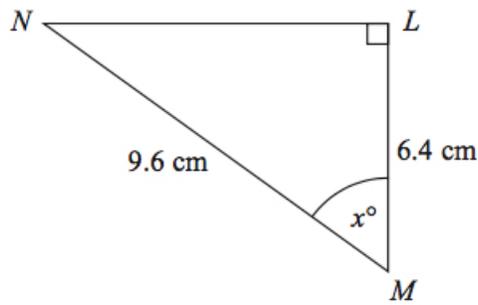Should Henry have a water meter?

Solution

Water meter:
$$28.20 + \frac{365 \times 0.9122 \times 180}{1\,000} = 88.131\,54$$
so he should have a water meter.

16. LMN is a right-angled triangle. (3)

$MN = 9.6$ cm.
$LM = 6.4$ cm.
Calculate the size of the angle marked $x°$.
Give your answer correct to 1 decimal place.

Solution

$$\cos = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos x° = \frac{6.4}{9.6}$$
$$\Rightarrow x = 48.189\,685\,1 \text{ (FCD)}$$
$$\Rightarrow x = 48.2 \text{ (1 dp)}.$$

17. Liam invests £6 200 for 3 years in a savings account. (3)
He gets 2.5% per annum compound interest.
How much money will Liam have in his savings account at the end of 3 years?

Solution

$$6\,200 \times 1.025^3 = 6\,676.721\,875 \text{ (FCD)}$$
$$= \pounds 6\,676.72 \text{ (nearest penny)}.$$

18. The diagram shows a quadrilateral $ABCD$. (5)
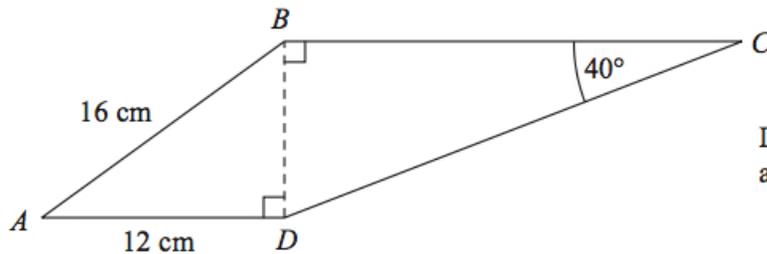
Diagram **NOT** accurately drawn

$AB = 16$ cm.
$AD = 12$ cm.
Angle $BCD = 40°$.
Angle $ADB = $ angle $CBD = 90°$.
Calculate the length of CD.
Give your answer correct to 3 significant figures.

Solution

$$BD = \sqrt{AB^2 - AD^2}$$
$$= \sqrt{16^2 - 12^2}$$
$$= \sqrt{112}$$
$$= 4\sqrt{7}.$$

Finally,

$$\text{hyp} = \frac{\text{opp}}{\sin} \Rightarrow CD = \frac{4\sqrt{7}}{\sin 40°}$$
$$\Rightarrow CD = 16.464\,233\,42 \text{ (FCD)}$$
$$\Rightarrow CD = 16.5 \text{ cm (3 sf)}.$$

19. (3)

$$p^2 = \frac{x - y}{xy}.$$

$x = 8.5 \times 10^9.$
$y = 4 \times 10^8.$
Find the value of p.
Give your answer in standard form correct to 2 significant figures.

Solution

$$p = \sqrt{\frac{x - y}{xy}}$$
$$= \sqrt{\frac{8.1 \times 10^9}{3.4 \times 10^{18}}}$$
$$= 4.880\,935\,301 \times 10^{-5} \text{ (FCD)}$$
$$= \underline{\underline{4.9 \times 10^{-5} \text{ (2 sf)}}}.$$

20. Make t the subject of the formula (3)

$$2(d - t) = 4t + 7.$$

Solution

$$2(d - t) = 4t + 7 \Rightarrow 2d - 2t = 4t + 7$$
$$\Rightarrow 2d - 7 = 6t$$
$$\Rightarrow \underline{\underline{t = \tfrac{1}{6}(2d - 7)}}.$$

21. Prove that (3)
$$(2n + 3)^2 - (2n - 3)^2$$
is a multiple of 8 for all positive integer values of n.

Solution

$$(2n + 3)^2 - (2n - 3)^2 = (4n^2 + 12n + 9) - (4n^2 - 12n + 9)$$
$$= 24n$$
$$= 8 \times 3n;$$

hence, it is true that is $(2n + 3)^2 - (2n - 3)^2$ a <u>multiple of 8</u> for all positive integer values of n.

22. Solve (3)
$$3x^2 - 4x - 2 = 0.$$

Give your solutions correct to 3 significant figures.

Solution

$a = 3$, $b = -4$, and $c = -2$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{4 \pm \sqrt{(-4)^2 - 4 \times 3 \times (-2)}}{2 \times 3}$$
$$= \frac{4 \pm \sqrt{40}}{6}$$
$$= -0.387\,425\,886\,7 \text{ or } 1.720\,759\,22 \text{ (FCD)}$$
$$= \underline{\underline{-0.387 \text{ or } 1.72 \text{ (3 sf)}}}.$$

23. (a) Max wants to take a random sample of students from his year group. (2)
    (i) Explain what is meant by a random sample.

    Solution
    It means every child has an <u>equal chance</u> of being picked.

    (ii) Describe a method Max could use to take his random sample.

    Solution
    E.g., he could number the students $1, \ldots, n$ and could use a random number generator on his calculator.

(b) The table below shows the numbers of students in 5 year groups at a school. (2)

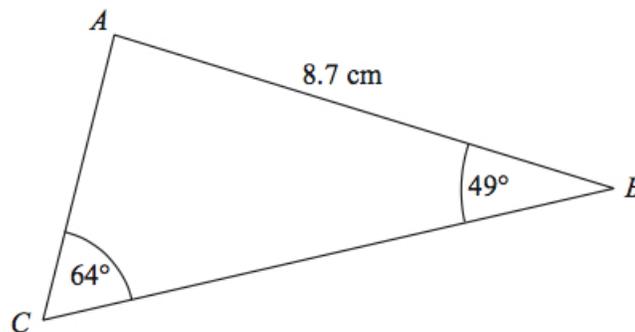| Year	Number of students
9	239
10	257
11	248
12	190
13	206

Lisa takes a stratified sample of 100 students by year group.
Work out the number of students from Year 9 she has in her sample.

Solution

There are $1\,140$ students in total and so the number of students from Year 9 she has in her sample is

$$\frac{239}{1\,140} \times 100 = 20.964\ldots$$
$$= \underline{\underline{21}}.$$

24. ABC is a triangle. (5)

$AB = 8.7$ cm.
Angle $ABC = 49°$.
Angle $ACB = 64°$.
Calculate the area of triangle ABC.
Give your answer correct to 3 significant figures.

Solution

First,
$$\angle BAC = 180 - 64 - 49 = 67°$$

and, second,
$$\frac{AC}{\sin 49°} = \frac{8.7}{\sin 64°} \Rightarrow AC = \frac{8.7 \sin 49°}{\sin 64°}$$
$$\Rightarrow AC = 7.305\,314\,688 \text{ (FCD)}.$$

Finally,
$$\text{area} = \tfrac{1}{2} \times 7.305\ldots \times 8.7 \times \sin 67°$$
$$= 29.251\,912\,67 \text{ (FCD)}$$
$$= \underline{\underline{29.3 \text{ cm}^2 \text{ (3 sf)}}}.$$

25. Carolyn has 20 biscuits in a tin. (4)
    She has 12 plain biscuits, 5 chocolate biscuits, and 3 ginger biscuits.
    Carolyn takes at random two biscuits from the tin.
    Work out the probability that the two biscuits were **not** the same type.
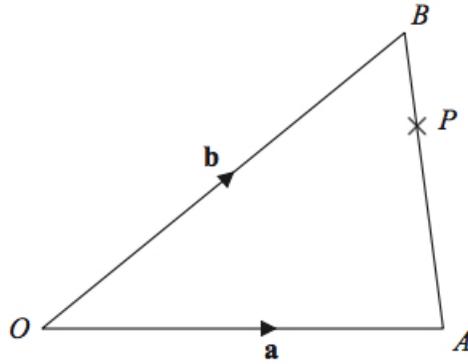
Solution

$$\text{P(same type)} = \left(\tfrac{12}{20} \times \tfrac{11}{19}\right) + \left(\tfrac{5}{20} \times \tfrac{4}{19}\right) + \left(\tfrac{3}{20} \times \tfrac{2}{19}\right)$$
$$= \tfrac{132}{380} + \tfrac{20}{380} + \tfrac{6}{380}$$
$$= \tfrac{79}{190}$$

and
$$\text{P(not the same type)} = 1 - \tfrac{79}{190} = \underline{\underline{\tfrac{111}{190}}}.$$

26. OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$.
$\overrightarrow{OB} = \mathbf{b}$.

(a) Find $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (1)

Solution

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$= \underline{\underline{-\mathbf{a} + \mathbf{b}}}.$$

P is the point on AB such that $AP : PB = 3 : 1$.

(b) Find $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (3)
   Give your answer in its simplest form.

Solution

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$
$$= \overrightarrow{OA} + \tfrac{3}{4}\overrightarrow{AB}$$
$$= \mathbf{a} + \tfrac{3}{4}(-\mathbf{a} + \mathbf{b})$$
$$= \mathbf{a} - \tfrac{3}{4}\mathbf{a} + \tfrac{3}{4}\mathbf{b}$$
$$= \tfrac{1}{4}\mathbf{a} + \tfrac{3}{4}\mathbf{b}$$
$$= \underline{\underline{\tfrac{1}{4}(\mathbf{a} + 3\mathbf{b})}}.$$