Dr Oliver Mathematics
GCSE Mathematics
2009 June Paper 3H: Non-Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. The two-way table gives some information about how 100 children travelled to school one day.

	Walk	Car	Other	Total
Boy	15		14	54
Girl		8	16	
Total	37			100

(a) Complete the two-way table. (3)

> **Solution**
>
> | | Walk | Car | Other | Total |
> |---|---|---|---|---|
> | Boy | 15 | $\underline{\underline{25}}$ | 14 | 54 |
> | Girl | $\underline{\underline{22}}$ | 8 | 16 | $\underline{\underline{46}}$ |
> | Total | 37 | $\underline{\underline{33}}$ | $\underline{\underline{30}}$ | 100 |

One of the children is picked at random.

(b) Write down the probability that this child walked to school that day. (1)

> **Solution**
> There are 37 children who walked and that the probability that this child walked to school that day is $\underline{0.37}$.

2. (a) Simplify $4x + 3y - 2x + 5y$. (2)

> **Solution**
> $$4x + 3y - 2x + 5y = \underline{\underline{2x + 8y}}.$$

Compasses cost c pence each.
Rulers cost r pence each.

(b) Write down an expression for the total cost, in pence, of 2 compasses and 4 rulers. (2)

Solution

$$\text{Cost} = \underline{\underline{2c + 4r}}.$$

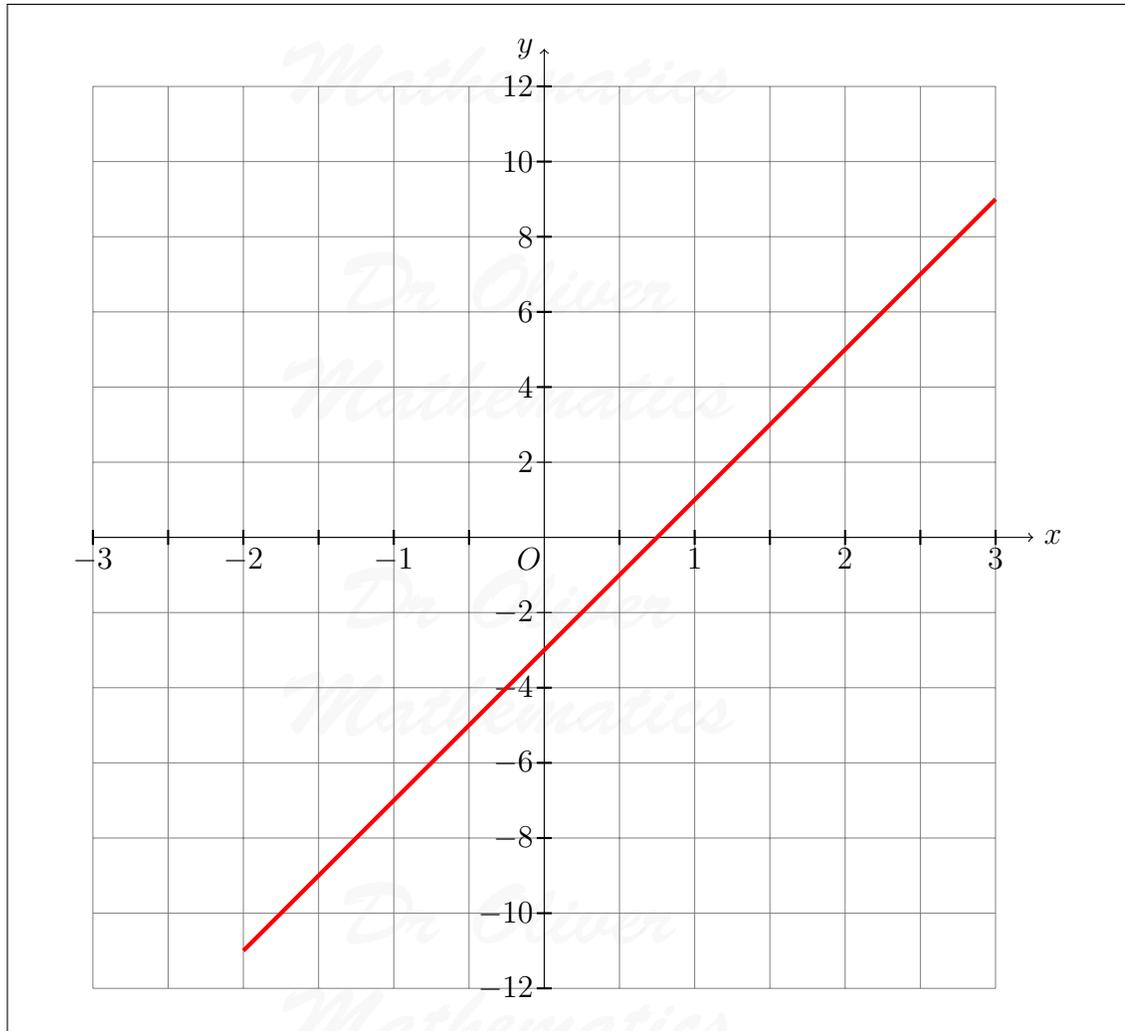3. (a) Complete the table of values for $y = 4x - 3$. (2)

x	-2	-1	0	1	2	3
y	-11		-3			9

Solution

x	-2	-1	0	1	2	3
y	-11	$\underline{\underline{-7}}$	-3	$\underline{\underline{1}}$	$\underline{\underline{5}}$	9

(b) On the grid, draw the graph of $y = 4x - 3$, for values of x from -2 to 3. (2)

Solution

2

4. $P = 4k - 10$.
   $P = 50$.

   (a) Work out the value of k. (2)

   > **Solution**
   >
   > $$4k - 10 = 50 \Rightarrow 4k = 60$$
   > $$\Rightarrow \underline{\underline{k = 15}}.$$

   $y = 4n - 3d$.
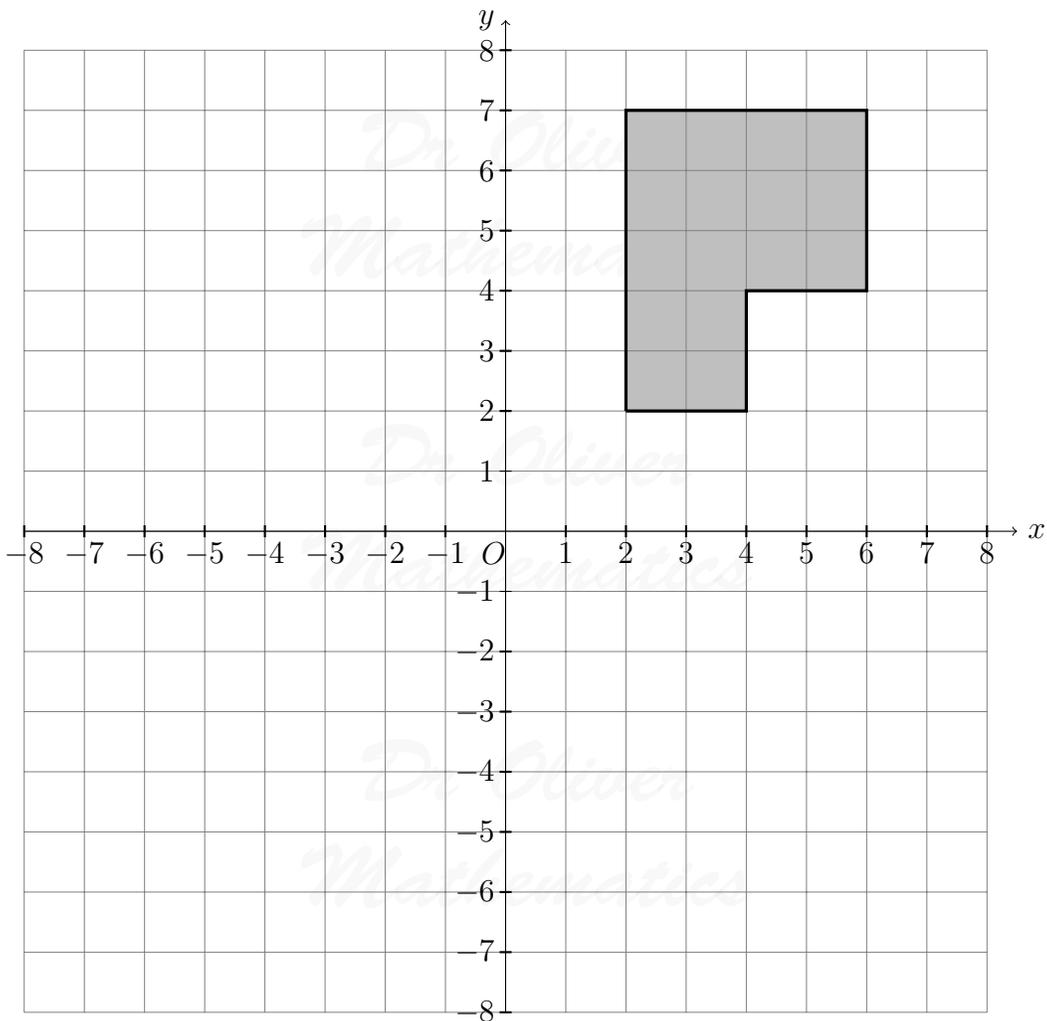 $n = 2$.
   $d = 5$.

(b) Work out the value of y. (2)

$$y = (4 \times 2) - (3 \times 5)$$
$$= 8 - 15$$
$$= \underline{\underline{-7}}.$$

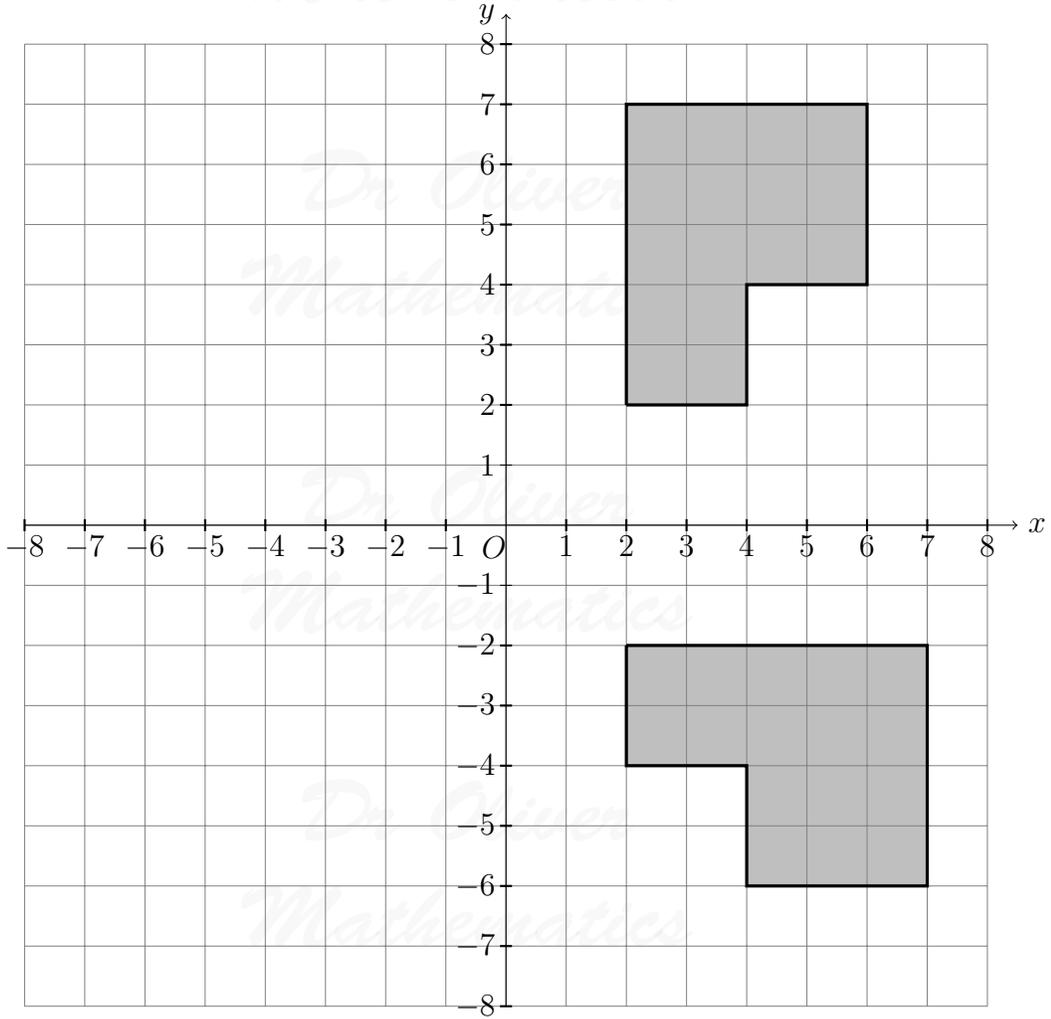5. (a) Rotate the shaded shape $90°$ clockwise about the point O. (2)

Solution
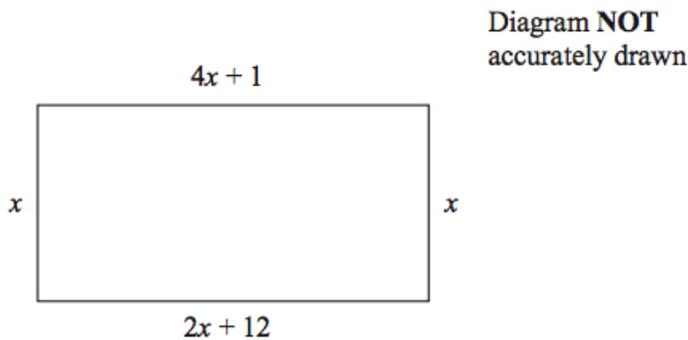
(b) Describe fully the single transformation that will map shape **P** onto shape **Q**. (2)

Solution

<u>Translation</u> by $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

6. The diagram shows a rectangle.
   All the measurements are in centimetres.

Diagram **NOT**
accurately drawn

$4x + 1$

x x

$2x + 12$

(a) Explain why $4x + 1 = 2x + 12$. (1)

Solution

They are the <u>opposite ends of a rectangle</u> and so they must be equal.

(b) Solve $4x + 1 = 2x + 12$. (2)

> **Solution**
>
> $$4x + 1 = 2x + 12 \Rightarrow 2x = 11$$
> $$\Rightarrow \underline{\underline{x = 5\tfrac{1}{2}}}.$$

(c) Use your answer to part (b) to work out the perimeter of the rectangle. (2)

> **Solution**
> Now,
> $$2x + 12 = (2 \times 5\tfrac{1}{2}) + 12 = 23$$
> and so the perimeter is
> $$2(23 + 5\tfrac{1}{2}) = 2 \times 28\tfrac{1}{2} = \underline{\underline{57 \text{ cm}}}.$$

7. Use the information that
$$322 \times 48 = 15\,456$$

to find the value of

(a) 3.22×4.8, (1)

> **Solution**
> Divide the 322 by 100 and divide the 4.8 by 10:
> $$3.22 \times 4.8 = \underline{\underline{15.456}}.$$

(b) 0.322×0.48, (1)

> **Solution**
> Divide the 322 by $1\,000$ and divide the 4.8 by 100:
> $$0.322 \times 0.48 = \underline{\underline{0.154\,56}}.$$

(c) $15\,456 \div 4.8$. (1)

> **Solution**
>
> Divide the 48 by 10 which means *multiplying* by 10 (why?):
>
> $$15\,456 \div 4.8 = \underline{3\,220}.$$

8. $2x^2 = 72$.

    (a) Find a value of x.   (2)

> **Solution**
>
> $$2x^2 = 72 \Rightarrow x^2 = 36$$
> $$\Rightarrow \underline{\underline{x = \pm 6}}.$$

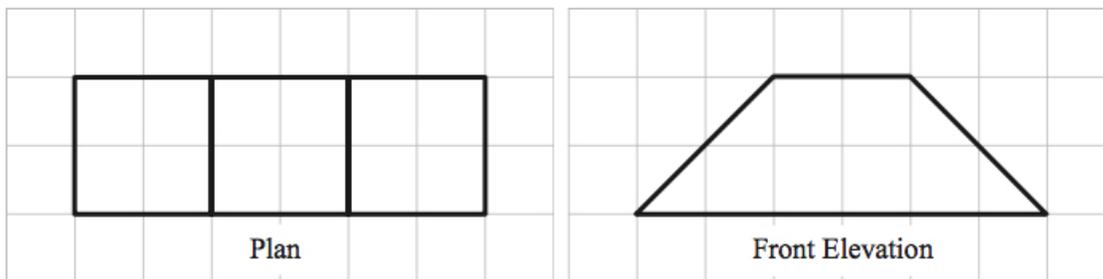   (b) Express 72 as a product of its prime factors.   (2)

> **Solution**
>
> | | 72 |
> |---|----|
> | 2 | 36 |
> | 2 | 18 |
> | 2 | 9 |
> | 3 | 3 |
> | 3 | 1 |
>
> and so
>
> $$72 = 2 \times 2 \times 2 \times 3 \times 3$$
> $$= \underline{\underline{2^3 \times 3^2}}.$$

9. Here are the plan and front elevation of a solid shape.

Plan

Front Elevation

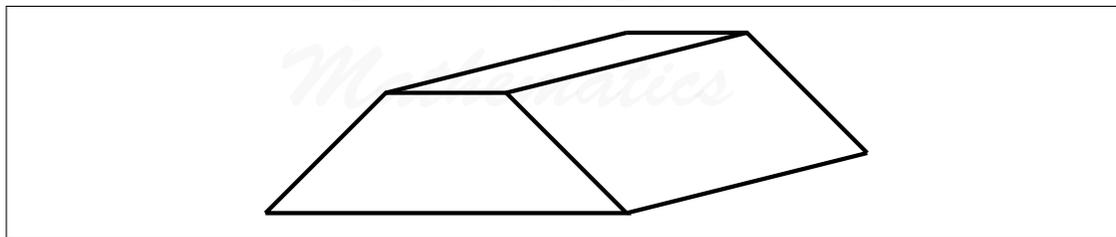(a) On the grid below, draw the side elevation of the solid shape. (2)

Solution

(b) Draw a sketch of the solid shape. (2)

Solution

E.g.,

9

10. There are 40 litres of water in a barrel. (3)
    The water flows out of the barrel at a rate of 125 millilitres per second.
    1 litre equals 1000 millilitres.
    Work out the time it takes for the barrel to empty completely.

    Solution

    $$40 \text{ litres} = 40\,000 \text{ millilitres}$$

    and the time it takes for the barrel to empty completely is

    $$\frac{40\,000}{125} = \frac{8\,000}{25} = \frac{1\,600}{5} = \underline{\underline{320 \text{ s}}}.$$

11. The length of a line is 63 centimetres, correct to the nearest centimetre.

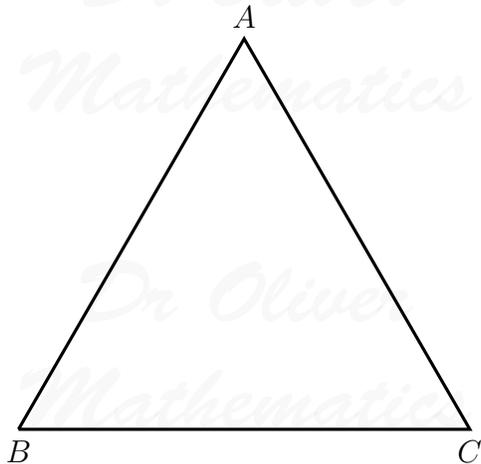   (a) Write down the **least** possible length of the line. (1)

    Solution
    $\underline{\underline{62.5 \text{ cm}}}$.

    (b) Write down the **greatest** possible length of the line. (1)

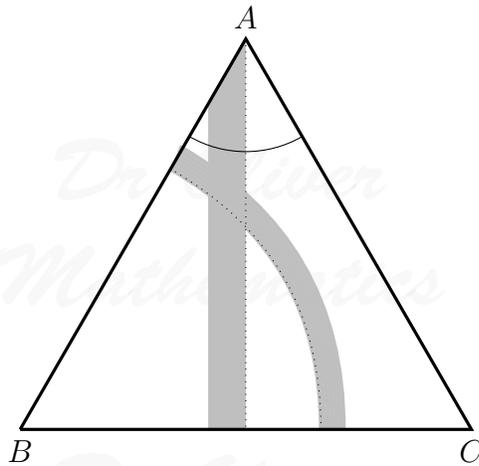   **Solution**
    $\underline{\underline{63.5 \text{ cm}}}$.

12. ABC is a triangle. (4)

10

Shade the region inside the triangle which is both less than 4 centimetres from the point B and closer to the line AC than the line AB.

Solution

13. Fred is going to take a survey of the magazines read by students.
He wants to design a questionnaire.

(a) Design a suitable question that he could use to find out what types of magazine students read. (2)

Solution

A suitable question with a time frame, e.g., "What magazines do you read? Tick all the appropriate box(es)."

> At least three exhaustive and non-overlapping tick boxes: for example, Never read, Computing, Football, Other (please specify).

Fred put the question below on his questionnaire.
'How many magazines have you read?'

A few	A lot

(b) Design a better question. (2)
You should include some response boxes.

> **Solution**
>
> A suitable question with a time frame, e.g., "Do you read magazines today/last week/last month? Tick the appropriate box."
> At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, 0, 1-3, 4-6, 7 or more.

14. Work out an estimate for the value of (3)

$$\frac{6.8 \times 191}{0.051}.$$

> **Solution**
>
> Use 1 significant figure:
> $$\frac{6.8 \times 191}{0.051} \approx \frac{7 \times 200}{0.05}$$
> $$= \frac{1\,400}{0.05}$$
> $$= \underline{\underline{2\,800}}.$$

15. (a) Write $64\,000$ in standard form. (1)

Solution

$$64\,000 = \underline{\underline{6.4 \times 10^5}}.$$

(b) Write 156×10^{-7} in standard form. (1)

Solution

$$156 \times 10^{-7} = \underline{\underline{1.56 \times 10^{-5}}}.$$

16. (a) Factorise fully $4x^2 - 6xy$. (2)

Solution

$$4x^2 - 6xy = \underline{\underline{2x(2x - 3y)}}.$$

(b) Factorise $x^2 + 5x - 6$. (2)

Solution

$$\left.\begin{array}{ll} \text{add to:} & +5 \\ \text{multiply to:} & -6 \end{array}\right\} + 6,\ -1$$

$$x^2 + 5x - 6 = \underline{\underline{(x + 6)(x - 1)}}.$$

17. Lucy did a survey about the amounts of money spent by 120 men during their summer holidays.
The cumulative frequency table gives some information about the amounts of money spent by the 120 men.

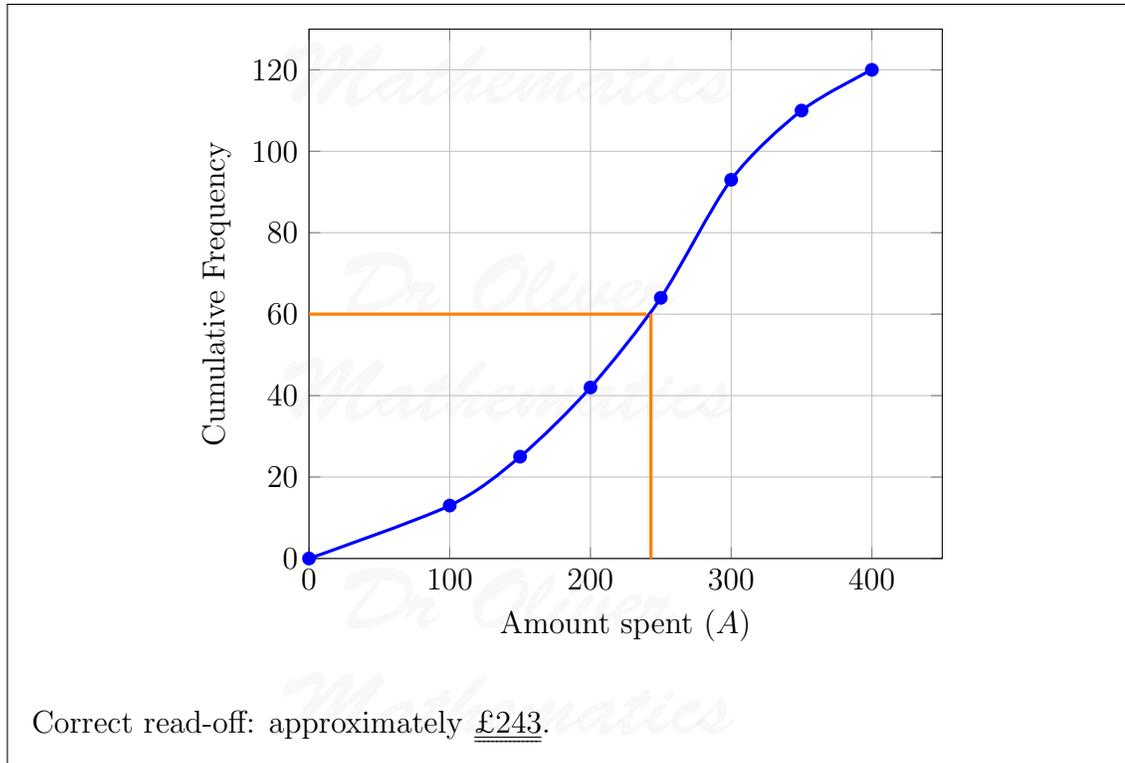| Amount (£A) spent	Cumulative frequency
$0 \leqslant A < 100$	13
$0 \leqslant A < 150$	25
$0 \leqslant A < 200$	42
$0 \leqslant A < 250$	64
$0 \leqslant A < 300$	93
$0 \leqslant A < 350$	110
$0 \leqslant A < 400$	120

(a) On the grid, draw a cumulative frequency diagram. (2)

13
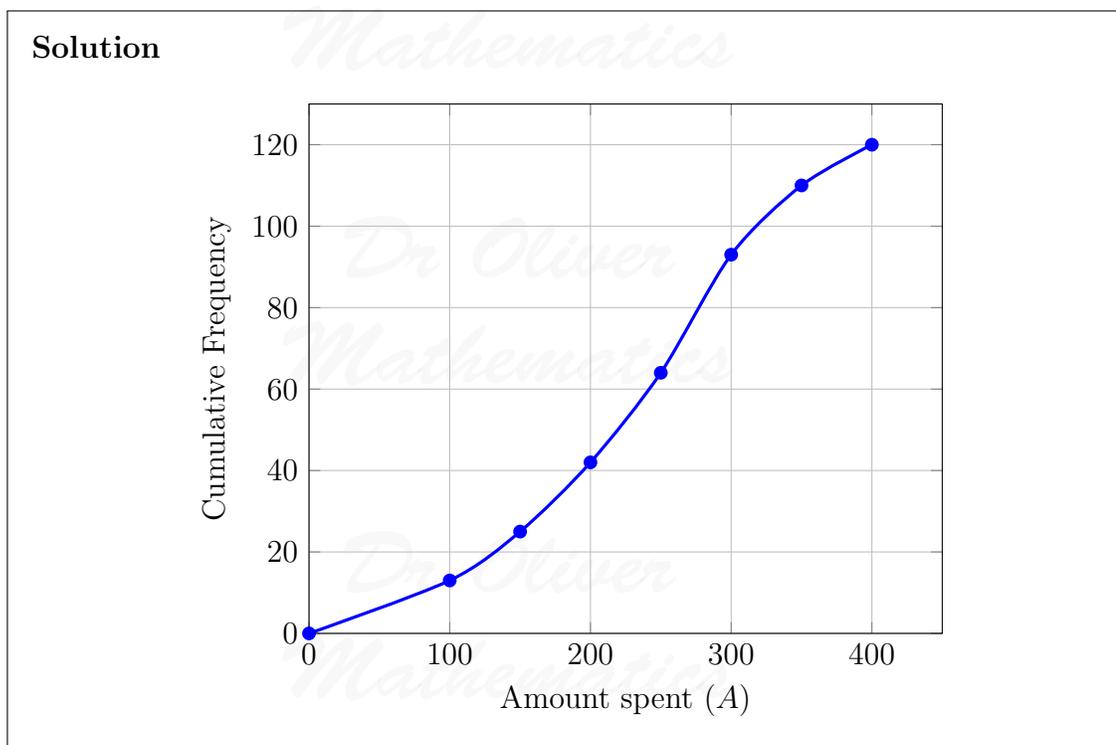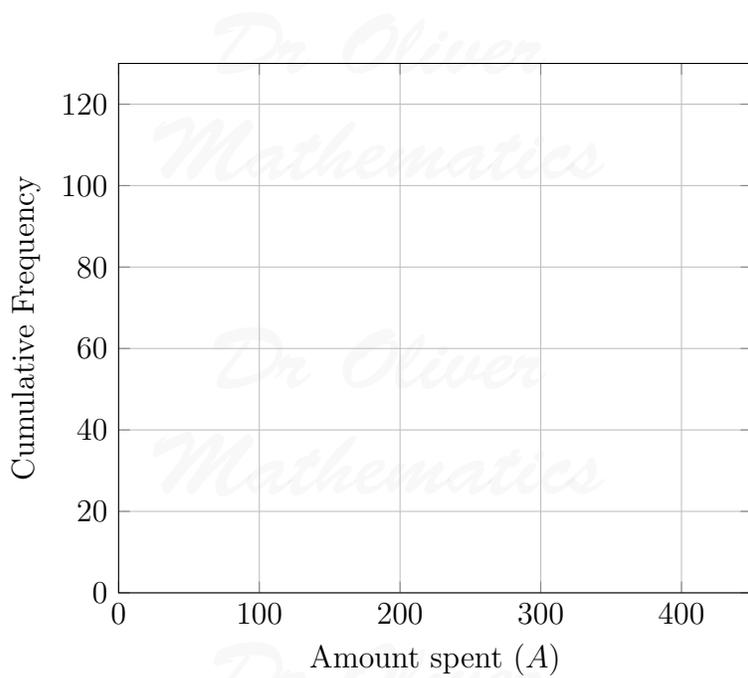
Solution

(b) Use your cumulative frequency diagram to estimate the median. (2)

Solution

14

Correct read-off: approximately £243.

A survey of the amounts of money spent by 200 women during their summer holidays gave a median of £205.

(c) Compare the amounts of money spent by the women with the amounts of money spent by the men. (1)

> **Solution**
>
> The median of the men is £243 whereas the median of the women is £205 and so the men spent more money.

18. The diagram shows a circle centre O.
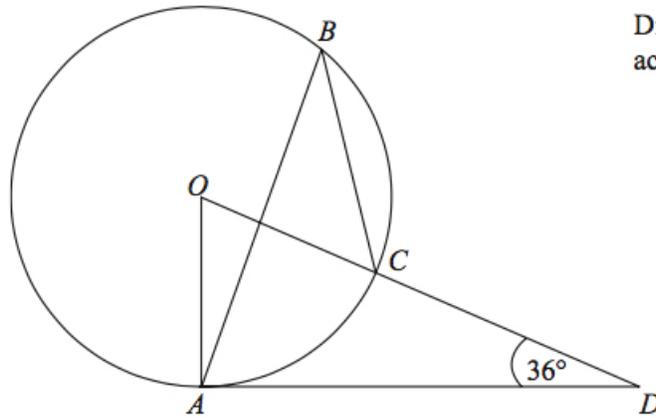 A, B, and C are points on the circumference.

Diagram **NOT** accurately drawn

DCO is a straight line.
DA is a tangent to the circle.
Angle $ADO = 36°$.

(a) Work out the size of angle AOD. (2)

> **Solution**
> It is
> $$180 - 90 - 36 = \underline{\underline{54°}}$$
> because $\angle OAD$ is a right angle.

(b) (i) Work out the size of angle ABC. (3)

> **Solution**
> $\underline{\underline{27°}}$.

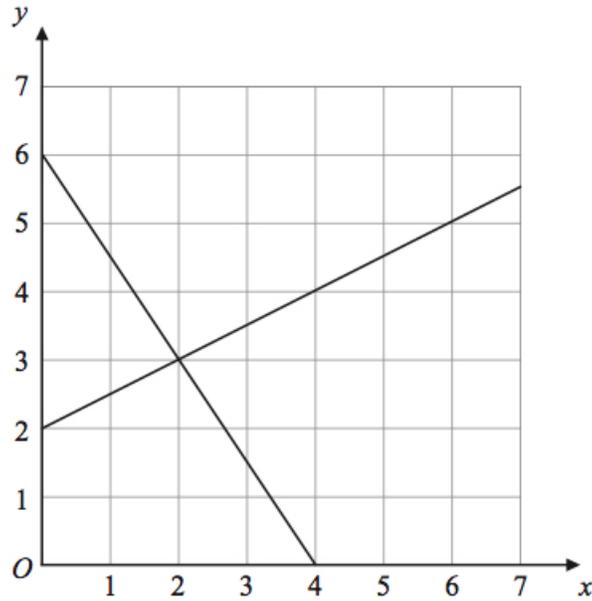(ii) Give a reason for your answer.

> **Solution**
> The angle is the centre is twice the angle at the circumference.

19. The diagram shows graphs of
$$y = \tfrac{1}{2}x + 2$$

and

$$2y + 3x = 12.$$

16

(a) Use the diagram to solve the simultaneous equations (1)

$$y = \tfrac{1}{2}x + 2$$
$$2y + 3x = 12.$$

> **Solution**
>
> $\underline{\underline{x = 2, y = 3}}$.

(b) Find an equation of the straight line which is parallel to the line $y = \tfrac{1}{2}x + 2$ and passes through the point $(0, 4)$. (2)

> **Solution**
>
> The equation is $y = \tfrac{1}{2}x + c$ for some c. Now,
>
> $$4 = \tfrac{1}{2} \times 0 + c \Rightarrow c = 4$$
>
> and the equation is $\underline{\underline{y = \tfrac{1}{2}x + 4}}$.

20. (a) Solve the inequality (2)

$$3t + 1 < t + 12.$$

Solution

$$3t + 1 < t + 12 \Rightarrow 2t < 11$$
$$\Rightarrow t < 5\tfrac{1}{2}.$$

(b) t is a whole number.　(1)
Write down the largest value of t that satisfies

$$3t + 1 < t + 12.$$

Solution

$\underline{\underline{5}}$.

21. M is directly proportional to L^3.　(4)
When $L = 2$, $M = 160$.
Find the value of M when $L = 3$.

Solution

$$M = kL^3$$

for some constant k. Now,

$$160 = k \times 2^3 \Rightarrow k = 20$$
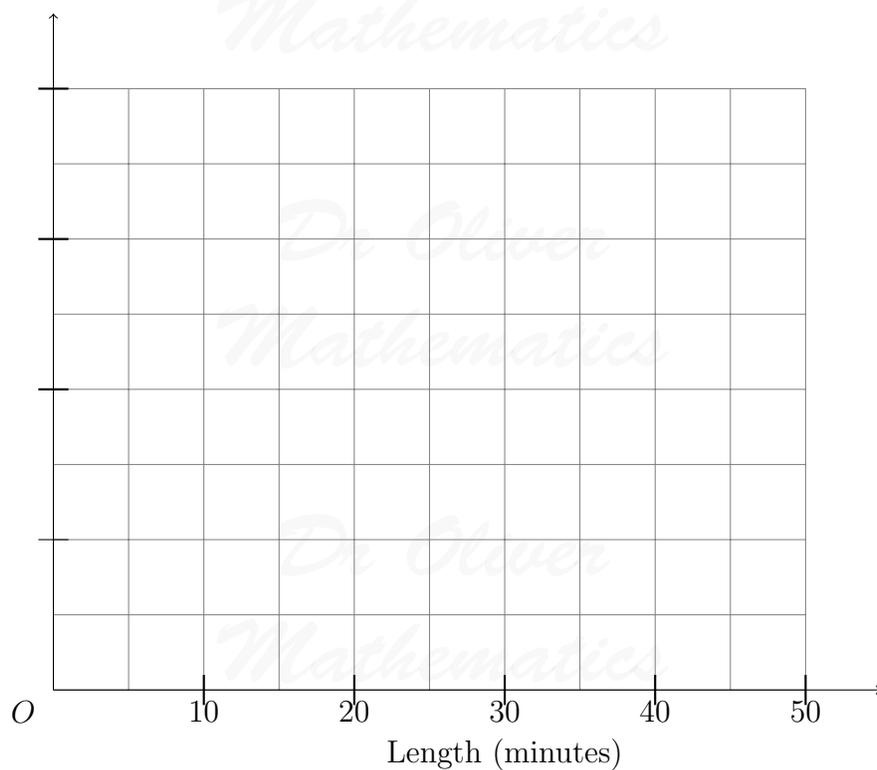
and

$$M = 20L^3.$$

Finally,

$$M = 20 \times 3^3 = 20 \times 27 = \underline{\underline{540}}.$$

22. A call centre receives 64 telephone calls one morning.　(4)
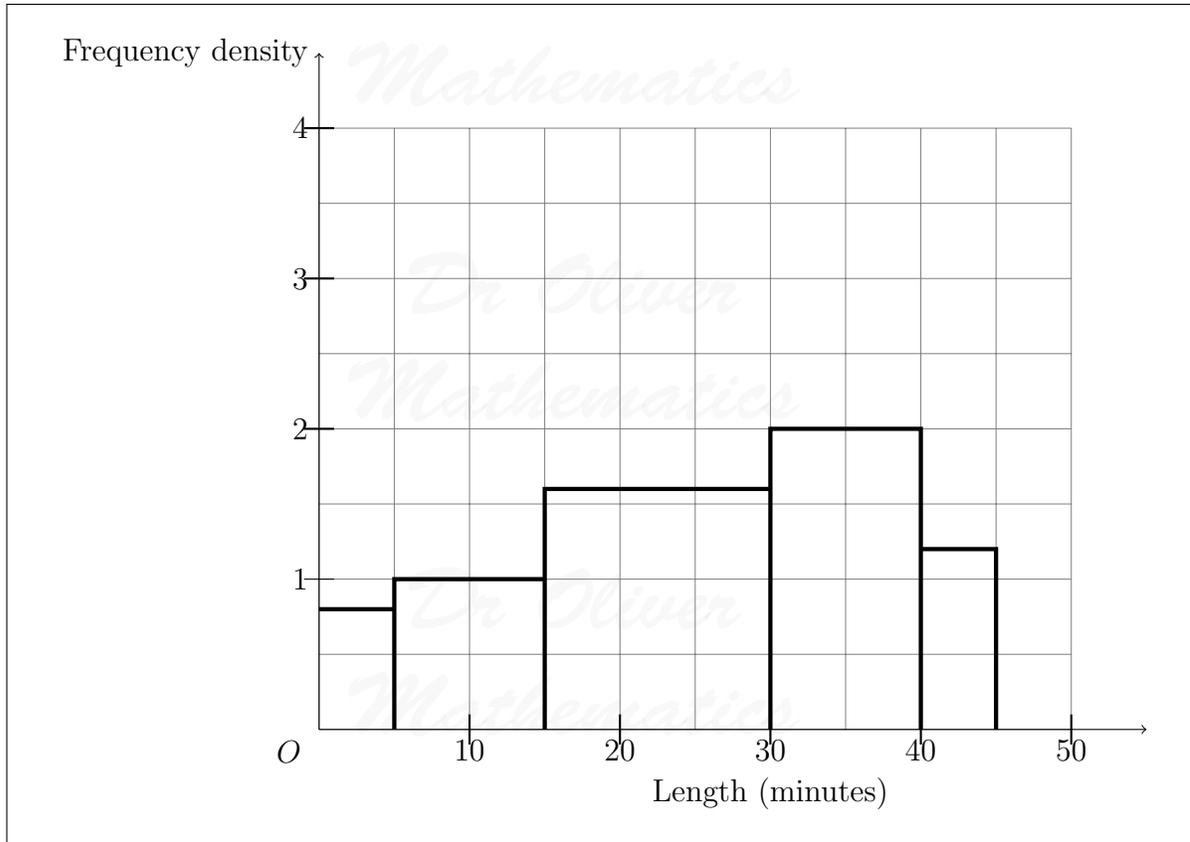The table gives information about the lengths, in minutes, of these telephone calls.

Length (x) minutes	Frequency
$0 < x \leqslant 5$	14
$5 < x \leqslant 15$	10
$15 < x \leqslant 30$	24
$30 < x \leqslant 40$	20
$40 < x \leqslant 45$	6

Draw a histogram for this information.

Length (minutes)

Solution

Length (x) minutes	Frequency	Width	Frequency density
$0 < x \leqslant 5$	4	5	$\frac{4}{5} = 0.8$
$5 < x \leqslant 15$	10	10	$\frac{10}{10} = 1$
$15 < x \leqslant 30$	24	15	$\frac{24}{15} = 1.6$
$30 < x \leqslant 40$	20	10	$\frac{20}{10} = 2$
$40 < x \leqslant 45$	6	5	$\frac{6}{5} = 1.2$

23. In a game of chess, a player can either win, draw, or lose.
    The probability that Vishi wins any game of chess is 0.5.
    The probability that Vishi draws any game of chess is 0.3.
    Vishi plays 2 games of chess.

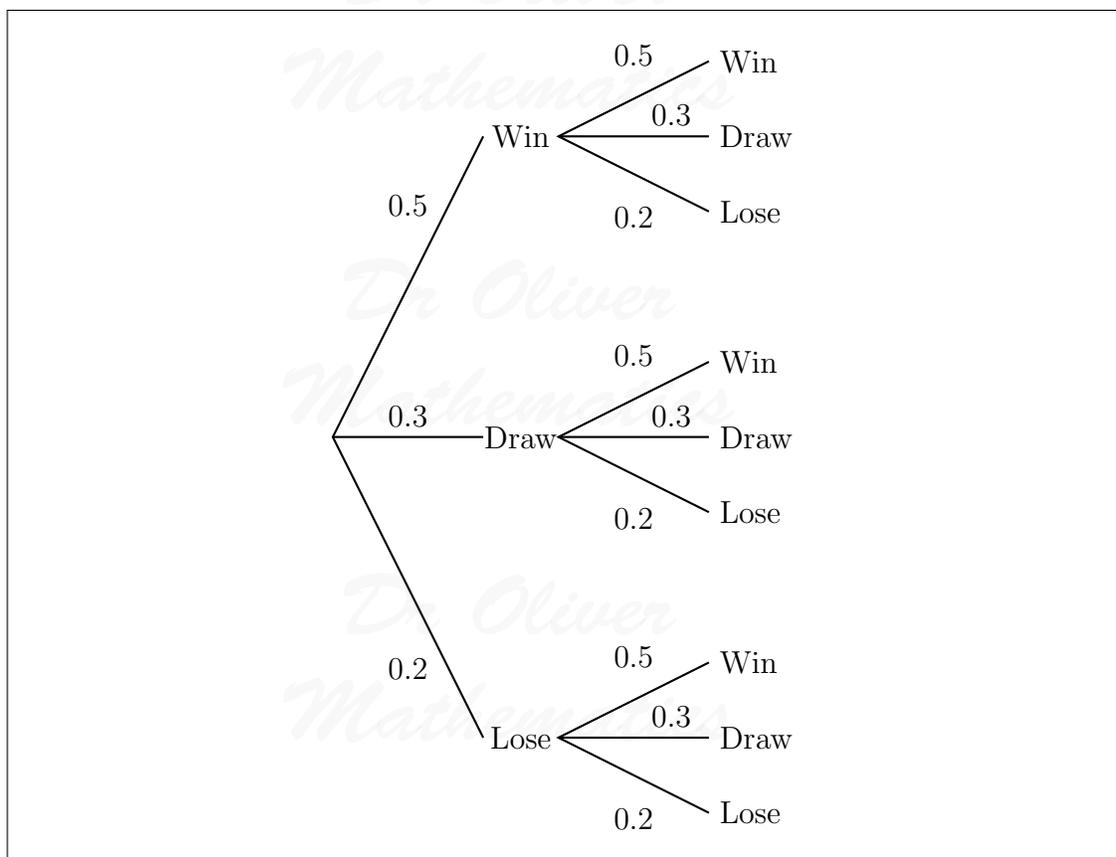   (a) Complete the probability tree diagram. (2)
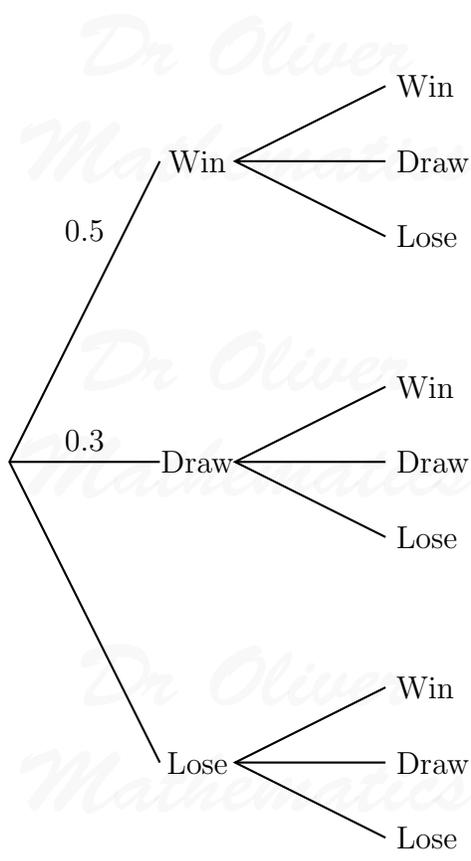
```
                                              Win

                                    Win       Draw

                                              Lose
                      0.5

                                              Win

                      0.3          Draw       Draw

                                              Lose


                                              Win

                      Lose         Draw

                                              Lose
```

Solution

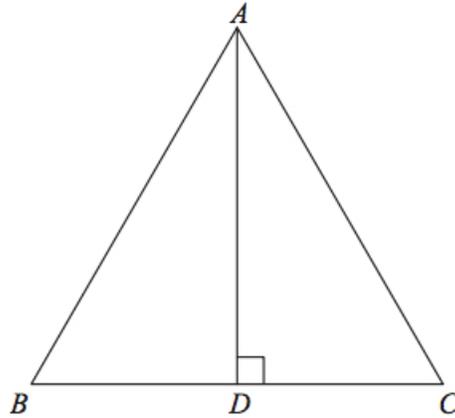(b) Work out the probability that Vishi will win both games. (2)

Solution
$$\mathrm{P}(WW) = 0.5 \times 0.5 = \underline{\underline{0.25}}.$$

24. ABC is an equilateral triangle.

Diagram **NOT**
accurately drawn

D lies on BC.
AD is perpendicular to BC.

(a) Prove that triangle ADC is congruent to triangle ADB. (3)

Solution

AD is shared.
$\angle ABC = \angle ACB$ (equilateral triangle).
$\angle ADB = \angle ADC$ (right angle).
So the triangle ADC is <u>congruent</u> to triangle ADB (AAS).

(b) Hence, prove that $BD = \frac{1}{2}AB$. (2)

Solution

$$BD = \tfrac{1}{2}BC \text{ (congruent)}$$
$$= \tfrac{1}{2}AB \text{ (equilateral triangle)},$$

as required.

25.
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

$u = 2\frac{1}{2}$ and $v = 3\frac{1}{3}$.

(a) Find the value of f. (3)

Solution

23

$$\frac{1}{2\frac{1}{2}} + \frac{1}{3\frac{1}{3}} = \frac{1}{f} \Rightarrow \frac{2}{5} + \frac{3}{10} = \frac{1}{f}$$
$$\Rightarrow \frac{7}{10} = \frac{1}{f}$$
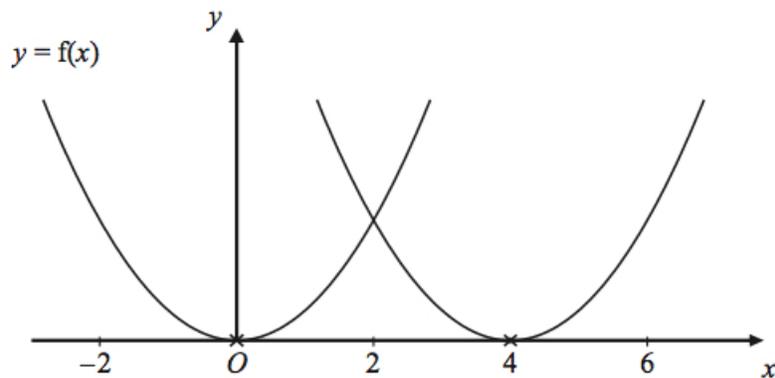$$\Rightarrow f = \frac{10}{7} \text{ or } 1\frac{3}{7}.$$

(b) Rearrange (2)

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

to make u the subject of the formula.
Give your answer in its simplest form.

Solution

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{1}{u} = \frac{1}{f} - \frac{1}{v}$$
$$\Rightarrow \frac{1}{u} = \frac{v - f}{fv}$$
$$\Rightarrow u = \frac{fv}{v - f}.$$

26. The curve with equation $y = \mathrm{f}(x)$ is translated so that the point at $(0,0)$ is mapped onto the point $(4,0)$.
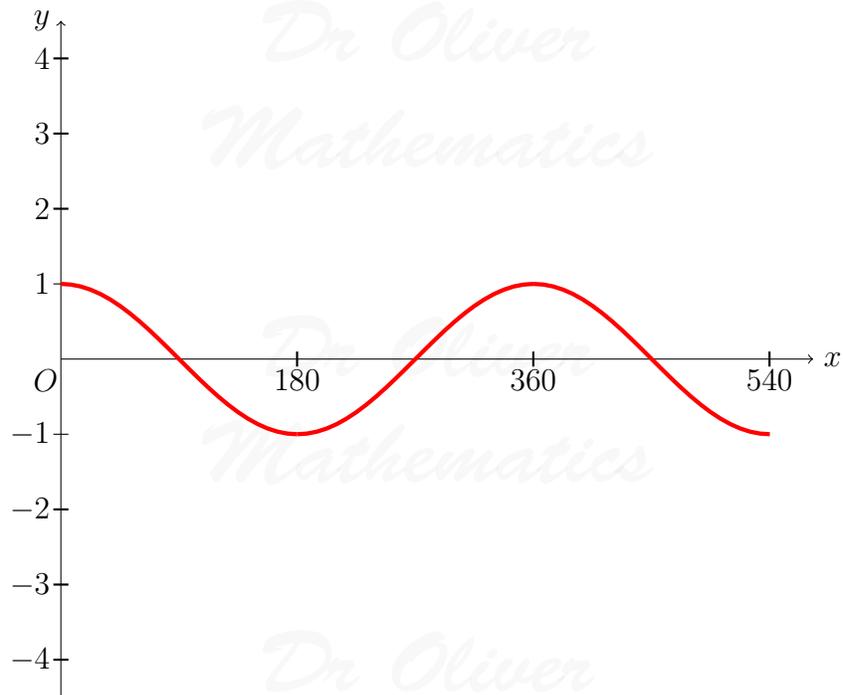

(a) Find an equation of the translated curve. (2)

The grid shows the graph of $y = \cos x°$ for values of x from 0 to 540.

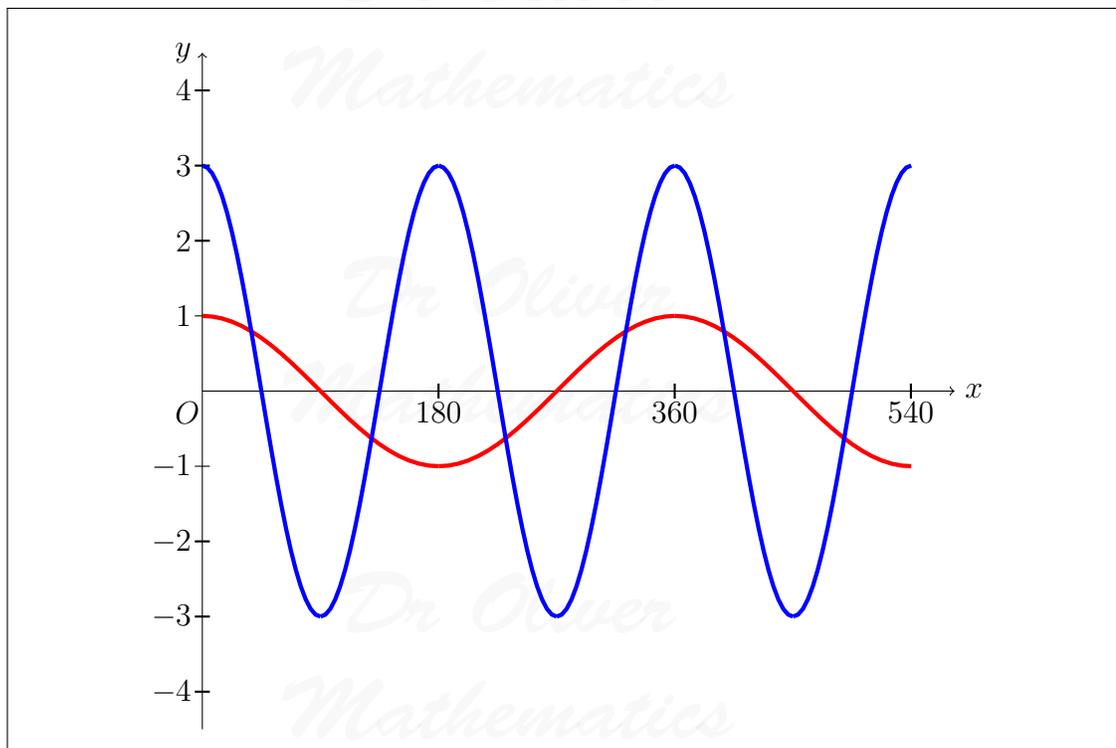(b) On the grid, sketch the graph of $y = 3\cos(2x°)$ for values of x from 0 to 540. (2)

Solution