Dr Oliver Mathematics
Mathematics: Higher
2014 Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 70.
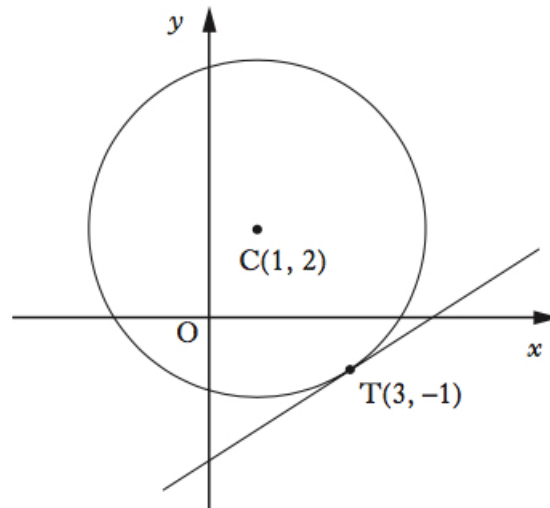You must write down all the stages in your working.

Section A

1. A sequence is defined by the recurrence relation (2)

$$u_{n+1} = \tfrac{1}{3}u_n + 1, \text{ with } u_2 = 15.$$

What is the value of u_4?

    A. $2\tfrac{1}{9}$

    B. $2\tfrac{1}{3}$

    C. 3

    D. 30

2. The diagram shows a circle with centre $C(1, 2)$ and the tangent at $T(3, -1)$. (2)

What is the gradient of this tangent?

    A. $\tfrac{1}{4}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 4

3. If (2)

$$\log_4 12 - \log_4 x = \log_4 6,$$

what is the value of x?

A. 2

B. 6

C. 18

D. 72

4. If (2)

$$3\sin x - 4\cos x$$

is written in the form

$$k\cos(x - a),$$

what are the values of $k\cos a$ and $k\sin a$?

A. $k\cos a = -3$ and $k\sin a = 4$

B. $k\cos a = 3$ and $k\sin a = -4$

C. $k\cos a = 4$ and $k\sin a = -3$

D. $k\cos a = -4$ and $k\sin a = 3$

5. Find (2)

$$\int (2x + 9)^5 \, dx.$$

A. $10(2x + 9)^4 + c$

B. $\frac{1}{4}(2x + 9)^4 + c$

C. $10(2x + 9)^6 + c$

D. $\frac{1}{12}(2x + 9)^6 + c$

6. Given that (2)

$$\mathbf{u} = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} \text{ and } \mathbf{v} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix},$$

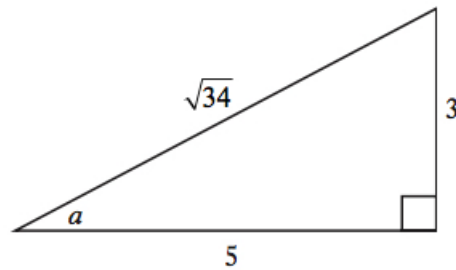find $2\mathbf{u} - 3\mathbf{v}$ in component form.

2

A. $\begin{pmatrix} -9 \\ 5 \\ -6 \end{pmatrix}$

B. $\begin{pmatrix} -9 \\ -1 \\ -4 \end{pmatrix}$

C. $\begin{pmatrix} -3 \\ -1 \\ 6 \end{pmatrix}$

D. $\begin{pmatrix} 11 \\ -5 \\ 4 \end{pmatrix}$

7. A right-angled triangle has sides and angles as shown in the diagram. (2)

What is the value of $\sin 2a$?

    A. $\frac{8}{17}$

    B. $\frac{3}{\sqrt{34}}$

    C. $\frac{15}{17}$

    D. $\frac{6}{\sqrt{34}}$

8. What is the derivative of (2)

$$(4 - 9x^4)^{\frac{1}{2}}?$$

A. $-\frac{9}{2}(4 - 9x^4)^{-\frac{1}{2}}$

B. $\frac{1}{2}(4 - 9x^4)^{-\frac{1}{2}}$

C. $2(4 - 9x^4)^{-\frac{1}{2}}$

D. $-18x^3(4 - 9x^4)^{-\frac{1}{2}}$

9. (2)

$$\sin x + \sqrt{3}\cos x$$

cvan be written as

$$2\cos(x - \tfrac{1}{6}\pi).$$

The maximum value of $\sin x + \sqrt{3}\cos x$ is 2.
What is the maximum value of $5\sin 2x + 5\sqrt{3}\cos 2x$?

    A. 20

    B. 10

    C. 5

    D. 2

10. A sequence is defined by the recurrence relation (2)
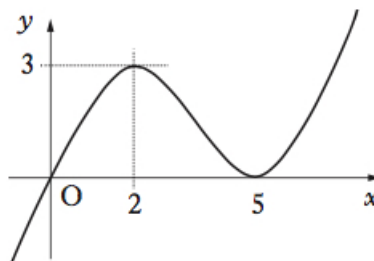
$$u_{n+1} = (k-2)u_n + 5, \text{ with } u_0 = 3.$$

For what values of k does this sequence have a limit as $n \to \infty$?

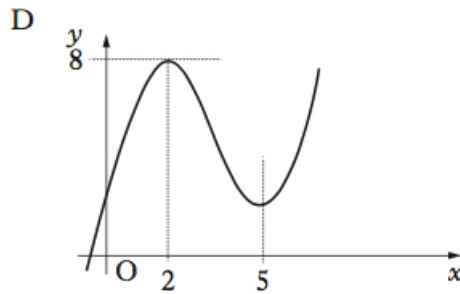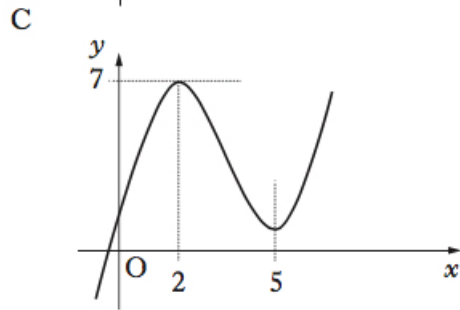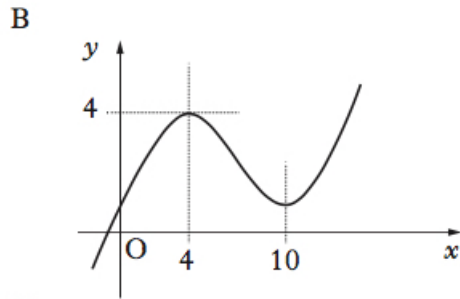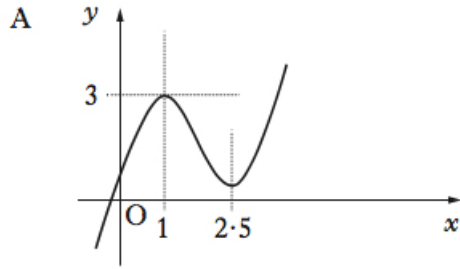   A. $-3 < k < -1$

    B. $-1 < k < 1$

    C. $1 < k < 3$

    D. $k < 3$

11. The diagram shows part of the graph of $y = \mathrm{f}(x)$. (2)

Which of the following diagrams could be the graph of $y = 2\,\mathrm{f}(x) + 1$?

A

B

C

D

12. A function f, defined on a suitable domain, is given by   (2)

$$f(x) = \frac{6x}{x^2 + 6x - 16}.$$

What restrictions are there on the domain of f?

    A. $x \neq -8$ or $x \neq 2$

    B. $x \neq -4$ or $x \neq 4$

    C. $x \neq 0$

D. $x \neq 10$ or $x \neq 16$

13. What is the value of (2)

$$\sin \tfrac{1}{3}\pi - \cos \tfrac{5}{4}\pi?$$

    A. $\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}$

    B. $\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}$

    C. $\frac{1}{2} - \frac{1}{\sqrt{2}}$

    D. $\frac{1}{2} + \frac{1}{\sqrt{2}}$

14. The vectors (2)

$$\mathbf{u} = \begin{pmatrix} 1 \\ k \\ k \end{pmatrix} \text{ and } \mathbf{v} = \begin{pmatrix} -6 \\ 2 \\ 5 \end{pmatrix}$$

are perpendicular.

What is the value of k?

    A. $-\frac{6}{7}$
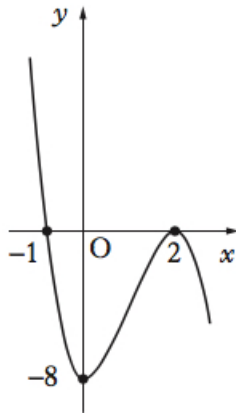
    B. -1

    C. 1

    D. $\frac{6}{7}$

15. The diagram shows a cubic curve passing through $(-1,0)$, $(2,0)$, and $(0,-8)$. (2)

What is the equation of the curve?

    A. $y = -2(x+1)^2(x+2)$

6

B. $y = -2(x+1)(x-2)^2$

C. $y = 4(x+1)^2(x-2)$

D. $y = -8(x+1)(x-2)^2$

16. The unit vectors **a** and **b** are such that   (2)

$$\mathbf{a}.\mathbf{b} = \tfrac{2}{3}.$$

Determine the value of

$$\mathbf{a}.(\mathbf{a} + 2\mathbf{b}).$$

    A. $\tfrac{2}{3}$

    B. $\tfrac{4}{3}$

    C. $\tfrac{7}{3}$

    D. 3

17.   (2)

$$3x^2 + 12x + 17$$

is expressed in the form

$$3(x+p)^2 + q.$$

What is the value of q?

    A. 1

    B. 5

    C. 17

    D. -19

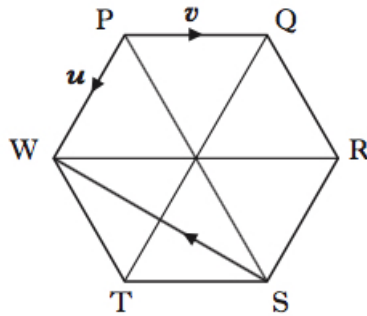18. What is the value of   (2)

$$1 - 2\sin^2 15°?$$

    A. $\tfrac{1}{2}$

    B. $\tfrac{3}{4}$

    C. $\tfrac{\sqrt{3}}{2}$

    D. $\tfrac{7}{8}$

19. The diagram shows a regular hexagon $PQRSTW$. $\quad$ (2)
$\overrightarrow{PW}$ and $\overrightarrow{PQ}$ represent vectors $\mathbf{u}$ and $\mathbf{v}$ respectively.

What is $\overrightarrow{SW}$ in terms of $\mathbf{u}$ and $\mathbf{v}$?

    A. $-\mathbf{u} - 2\mathbf{v}$

    B. $-\mathbf{u} - \mathbf{v}$

    C. $\mathbf{u} - \mathbf{v}$

    D. $\mathbf{u} + 2\mathbf{v}$

20. Evaluate $\quad$ (2)

$$2 - \log_5 \tfrac{1}{25}.$$

    A. -3

    B. 0

    C. $\frac{3}{2}$

    D. 4

Section B

21. A curve has equation $y = 3x^2 - x^3$.

    (a) Find the coordinates of the stationary points on this curve and determine their nature. $\quad$ (6)

    (b) State the coordinates of the points where the curve meets the coordinate axes and sketch the curve. $\quad$ (2)

22. For the polynomial $6x^3 + 7x^2 + ax + b$,

    • $(x + 1)$ is a factor, and

- 72 is the remainder when it is divided by $(x-2)$.

(a) Determine the values of a and b. (4)

(b) Hence factorise the polynomial completely. (3)

23. (a) Find P and Q, the points of intersection of the line (4)

$$y = 3x - 5$$

and the circle C_1 with equation

$$x^2 + y^2 + 2x - 4y - 15 = 0.$$

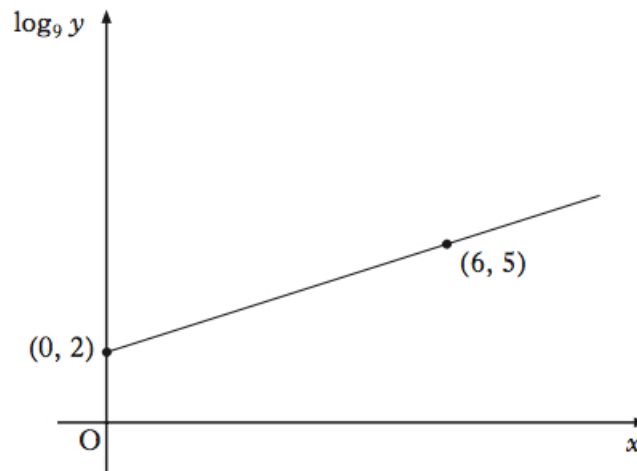T is the centre of C_1.

(b) Show that PT and QT are perpendicular. (3)

A second circle C_2 passes through P, Q, and T.

(c) Find the equation of C_2. (3)

24. Two variables, x and y, are related by the equation (5)

$$y = ka^x.$$

When $\log_9 y$ is plotted against x, a straight line passing through the points $(0, 2)$ and $(6, 5)$ is obtained, as shown in the diagram.

Find the values of k and a.