Dr Oliver Mathematics
GCSE Mathematics
2011 June Paper 3H: Non-Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. Here is a list of ingredients for making **10** Flapjacks. (3)

> **Ingredients for 10 Flapjacks**
>
> 80 g rolled oats
>
> 60 g butter
>
> 30 ml golden syrup
>
> 36 g light brown sugar

Work out the amount of each ingredient needed to make **15** Flapjacks.

Solution

Rolled oats: $1.5 \times 80 = \underline{\underline{120 \text{ g}}}$.
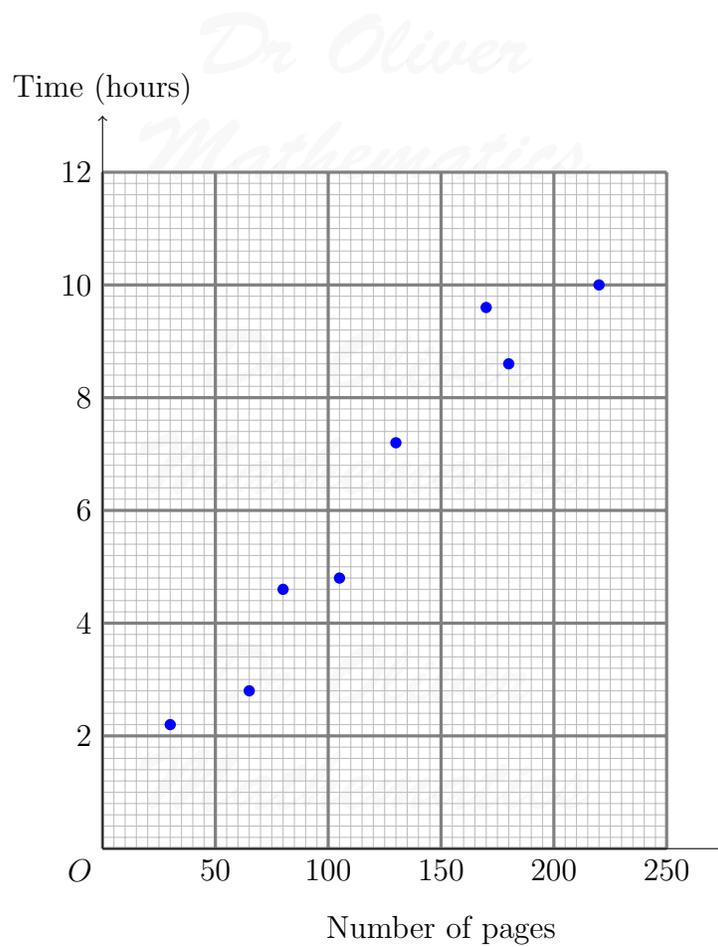
Butter: $1.5 \times 60 = \underline{\underline{90 \text{ g}}}$.

Golden syrup: $1.5 \times 30 = \underline{\underline{45 \text{ ml}}}$.

Brown sugar: $1.5 \times 36 = \underline{\underline{54 \text{ g}}}$.

2. Harriet reads eight books.
   For each book she recorded the number of pages and the time she takes to read it.
   The scatter graph shows information about her results.

Time (hours)

Number of pages

(a) Describe the relationship between the number of pages in a book and the time Harriet takes to read it. (1)

Solution

Positive correlation.
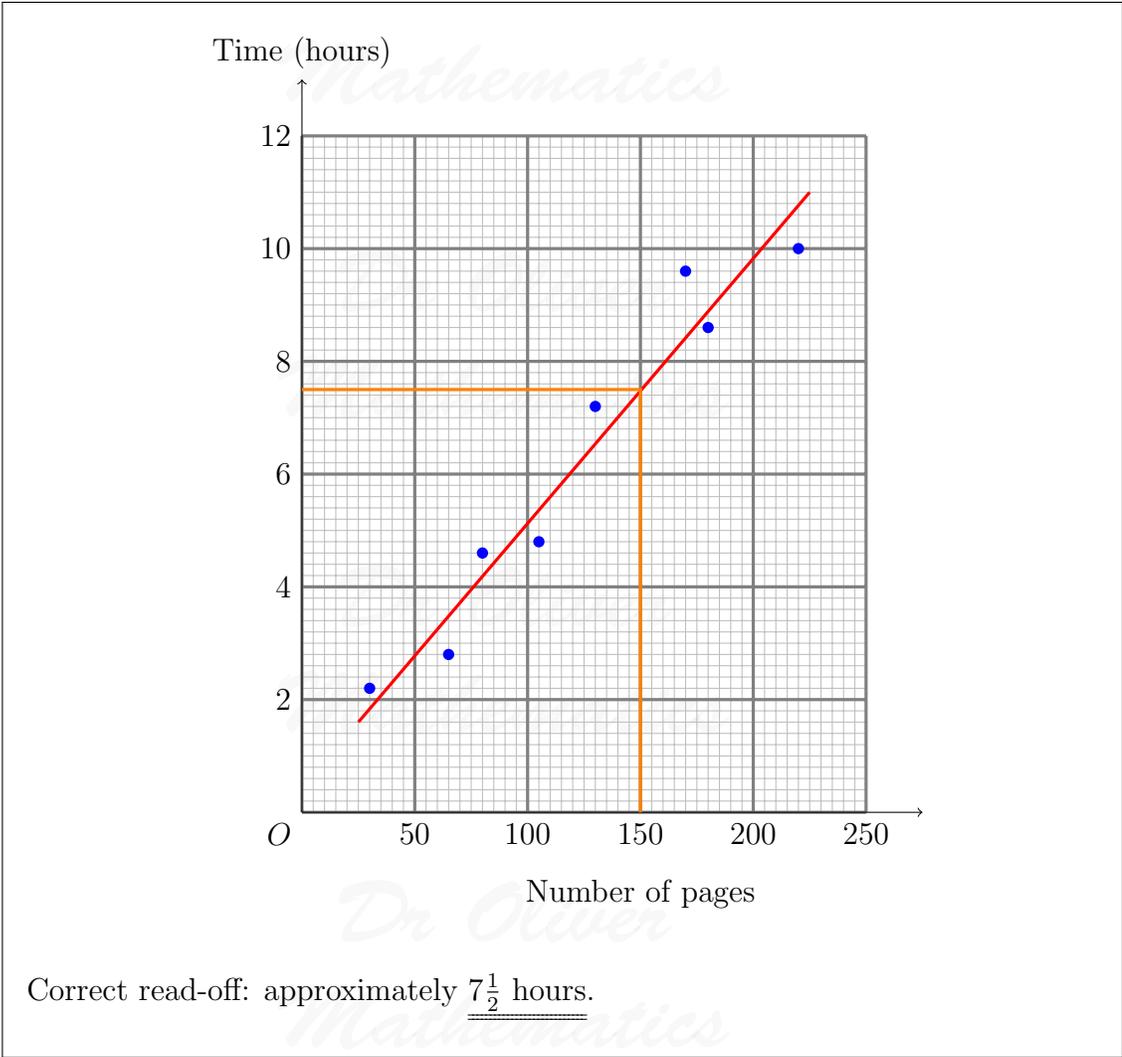
Harriet reads another book.
The book has 150 pages.
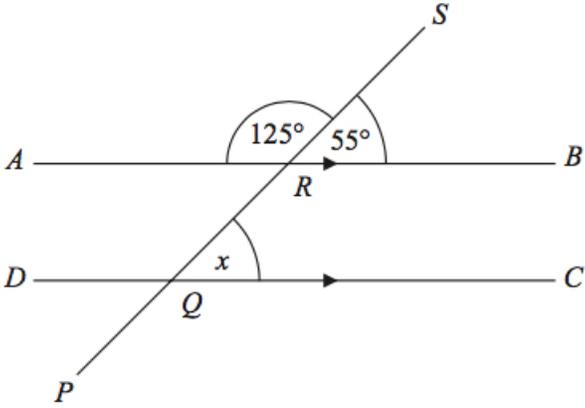
(b) Estimate the time it takes Harriet to read it. (2)

Solution

Draw the line of best fit:

2

Time (hours)

Number of pages

Correct read-off: approximately $7\frac{1}{2}$ hours.

3. ARB is parallel to DQC.

$PQRS$ is a straight line.

Angle $SRB = 55°$.

(a) Find the size of the angle marked x. (1)

Solution

$\underline{\underline{55°}}$.

(b) Give a reason for your answer. (1)

Solution

E.g., <u>corresponding angles</u>.

4. Work out an estimate for (3)

$$\frac{7.19 \times 19.7}{0.46}.$$

Solution

Round to 1 significant figure:

$$\frac{7.19 \times 19.7}{0.46} \approx \frac{7 \times 20}{0.5}$$
$$= \frac{140}{0.5}$$
$$= \underline{\underline{280}}.$$

5.

$$h = 5t^2 + 2.$$

(a) (i) Work out the value of h when $t = -2$. (3)

Solution

$$h = 5(-2)^2 + 2$$
$$= 5 \times 4 + 2$$
$$= \underline{\underline{22}}.$$

4

(ii) Work out a value of t when $h = 47$.

Solution

$$47 = 5t^2 + 2 \Rightarrow 5t^2 = 45$$
$$\Rightarrow t^2 = 9$$
$$\Rightarrow \underline{\underline{t = \pm 3}}.$$

(b) $-1 \leqslant n < 4$. (2)
n is an integer.
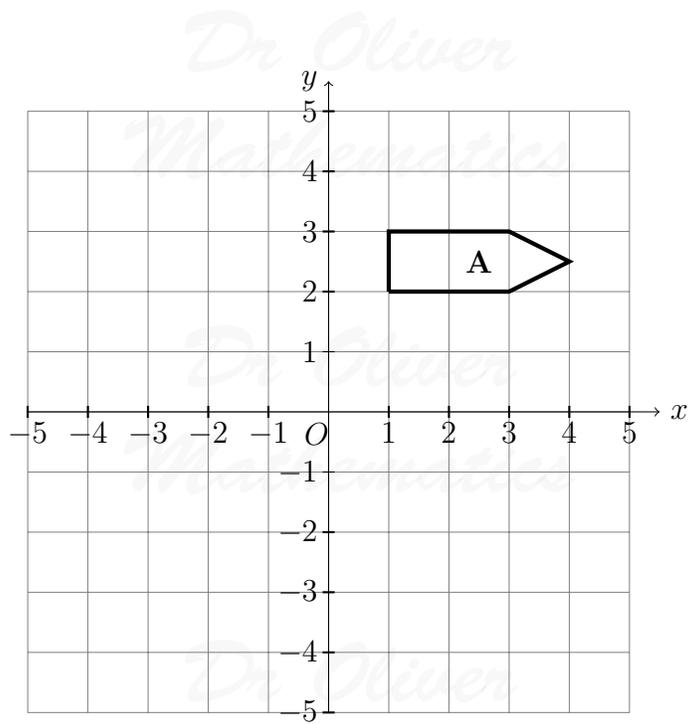Write down all the possible values of n.

Solution

$\underline{\underline{-1, 0, 1, 2, 3}}$.

6. Each exterior angle of a regular polygon is $30°$. (2)
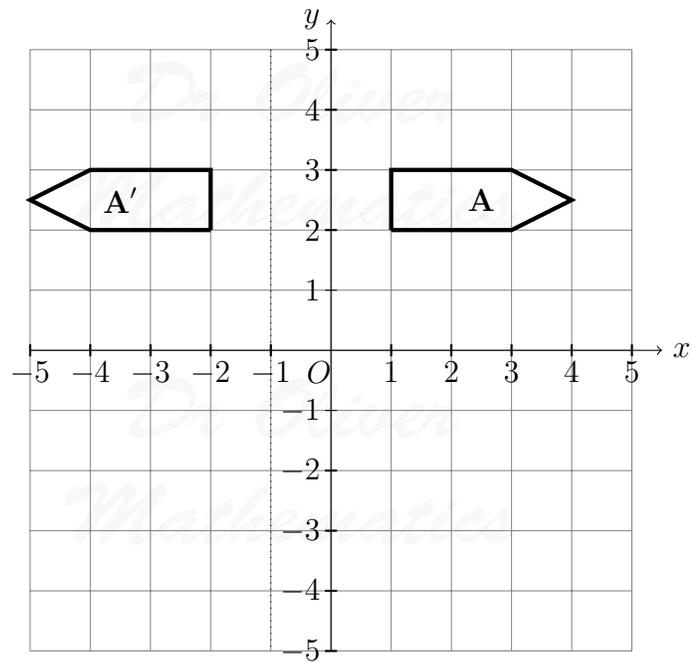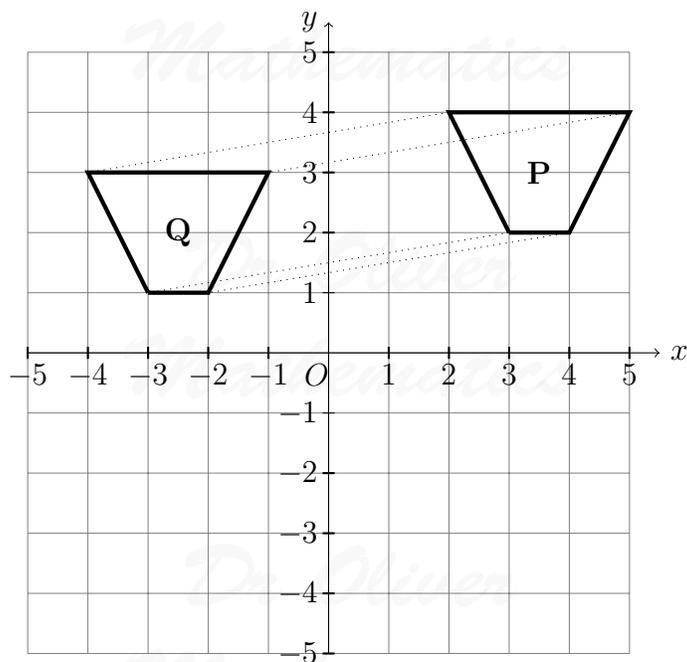Work out the number of sides of the polygon.

Solution

$$\frac{360}{30} = \underline{\underline{12 \text{ sides}}}.$$

7. (a) On the grid above, reflect shape **A** in the line $x = -1$. (2)

Solution

(b) Describe fully the single transformation that will map shape **P** onto shape **Q**. (2)

> **Solution**
>
> <u>Translation</u>, by $\begin{pmatrix} -6 \\ -1 \end{pmatrix}$.

8. Sophie wants to find out the amount of time people exercise.
   She will use a questionnaire.

   (a) Design a suitable question for Sophie to use in her questionnaire. (2)
      You must include some response boxes.

   > **Solution**
   >
   > A suitable question with a time frame, e.g., "Did you exercise today/last week/last month? Tick the appropriate box."
   > At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, Did not exercise, $0 < t$ mins < 15, $15 < t$ mins < 45, t mins $\geqslant 45$.

   Sophie asks the people at her swimming pool to complete her questionnaire.
   This may **not** be a suitable sample.

   (b) Give a reason why. (1)

> **Solution**
> E.g., it is a <u>biased</u> sample, it is <u>one type</u> of exercise (weight training, running, etc).

9. The nth term of a number sequence is given by $3n + 1$.

   (a) Work out the first **two** terms of the number sequence. (1)

   > **Solution**
   > $3 \times 1 + 1 = \underline{\underline{4}}$.
   > $3 \times 2 + 1 = \underline{\underline{7}}$.

   Here are the first four terms of another number sequence:

   $$1 \quad 5 \quad 9 \quad 13.$$

   (b) Find, in terms of n, an expression for the nth term of this number sequence. (2)

   > **Solution**
   > Let the
   > $$n\text{th term} = an + b.$$
   >
   > $$\begin{array}{cccc} 1 & 5 & 9 & 13 \\ \quad 4 & \quad 4 & \quad 4 & \end{array}$$
   >
   > $$\begin{array}{cccc} a + b & 2a + b & 3a + b & 4a + b \\ \quad a & \quad a & \quad a & \end{array}$$
   >
   > We compare terms:
   > $$a = 4$$
   > and
   > $$a + b = 1 \Rightarrow 4 + b = 1$$
   > $$\Rightarrow b = -3.$$
   > Hence,
   > $$n\text{th term} = \underline{\underline{4n - 3}}.$$

10. The diagram shows the cross-section of a solid prism. (5)
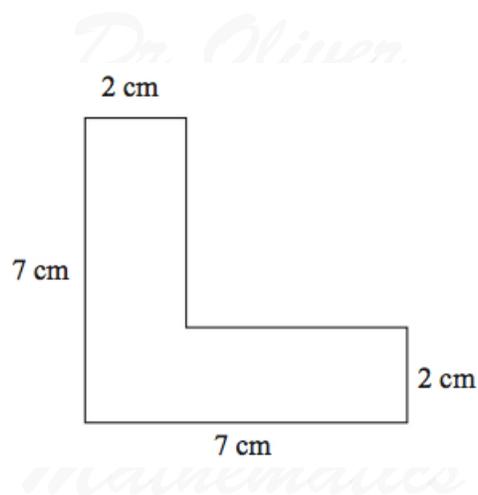
The length of the prism is 2 m.
The prism is made from metal.
The density of the metal is 8 grams per cm^3.
Work out the mass of the prism.

Solution

$$\text{Area of the cross-section} = 7 \times 7 - 5 \times 5$$
$$= 49 - 25$$
$$= 24 \text{ cm}^2$$

and

$$\text{volume} = 24 \times 200$$
$$= 4\,800 \text{ cm}^3.$$

Finally,

$$\text{mass} = 4\,800 \times 8$$
$$= 4\,800 \times 10 - 4\,800 \times 2$$
$$= 48\,000 - 9\,600$$
$$= \underline{\underline{38\,400 \text{ g}}}.$$

11. Peter, Tarish, and Ben share £54.   (3)
    Tarish gets three times as much money as Peter.
    Ben gets twice as much money as Tarish.
    How much money does Ben get?

Solution

$T = 3P$ and $B = 2T$. Now,

$$P + T + B = 54 \Rightarrow \tfrac{1}{3}T + T + 2T = 54$$
$$\Rightarrow \tfrac{10}{3}T = 54$$
$$\Rightarrow \tfrac{1}{3}T = 5.4$$
$$\Rightarrow T = 16.2$$
$$\Rightarrow \underline{\underline{B = £32.40}}.$$

12. (a) Simplify (2)
   (i) $w^6 \times w^4$,

   Solution
   $$w^6 \times w^4 = \underline{\underline{w^{10}}}.$$

   (ii) $h^8 \div h^3$.

   Solution
   $$\frac{h^8}{h^3} = \underline{\underline{h^5}}.$$

   (b) Simplify completely (2)
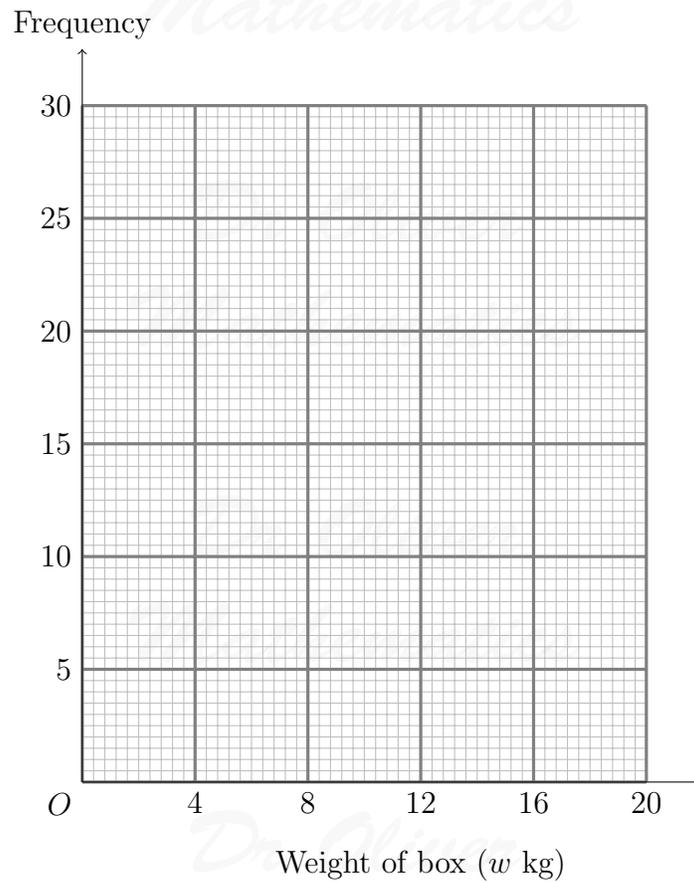 $$\frac{12xy^3}{3x^2y^3}.$$

   Solution
   $$\frac{12xy^3}{3x^2y^3} = \underline{\underline{\frac{4}{x}}} \text{ or } 4x^{-1}.$$

13. The table shows some information about the weights, in kg, of 100 boxes. (2)
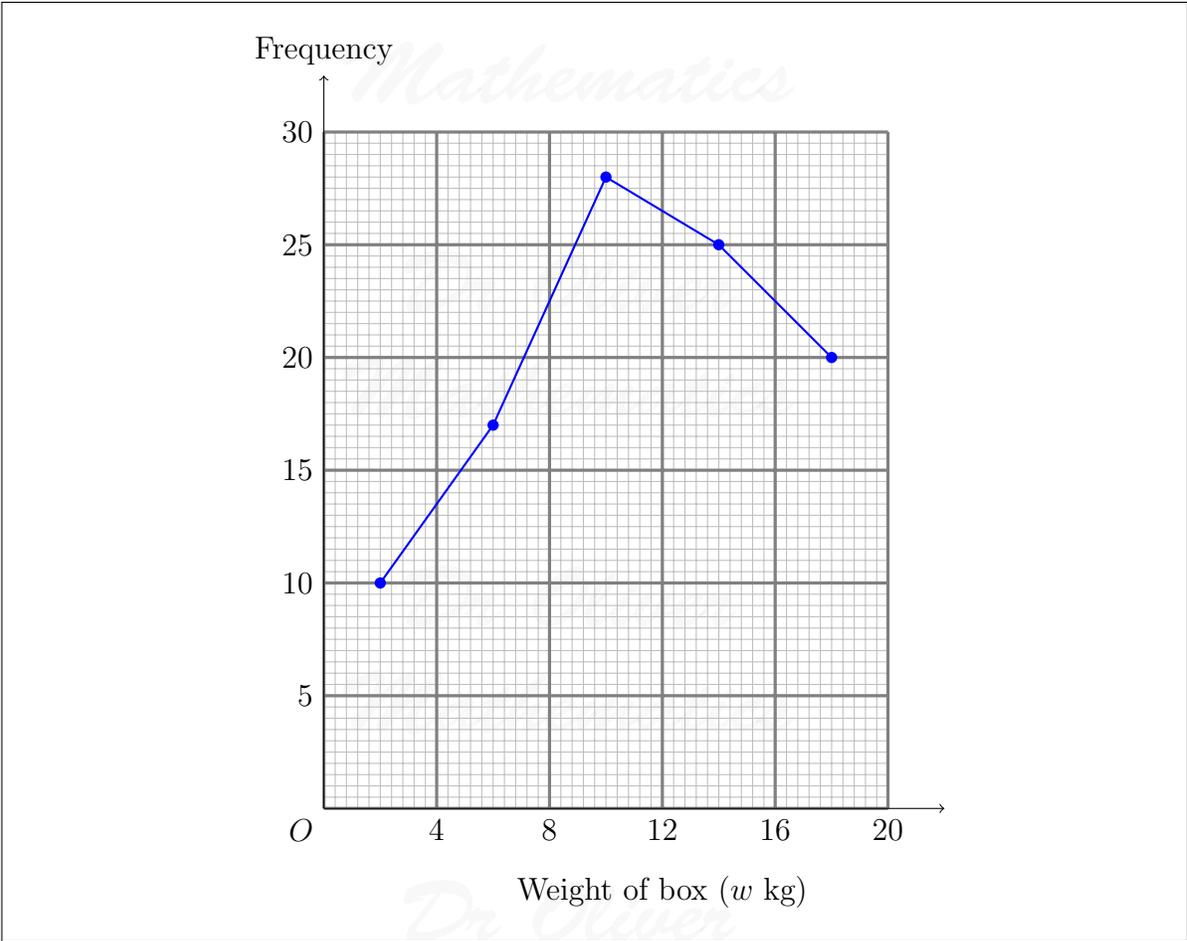
Weight of box (w kg)	Frequency
$0 < w \leqslant 4$	10
$4 < w \leqslant 8$	17
$8 < w \leqslant 12$	28
$12 < w \leqslant 16$	25
$16 < w \leqslant 20$	20

Draw a frequency polygon to show this information.

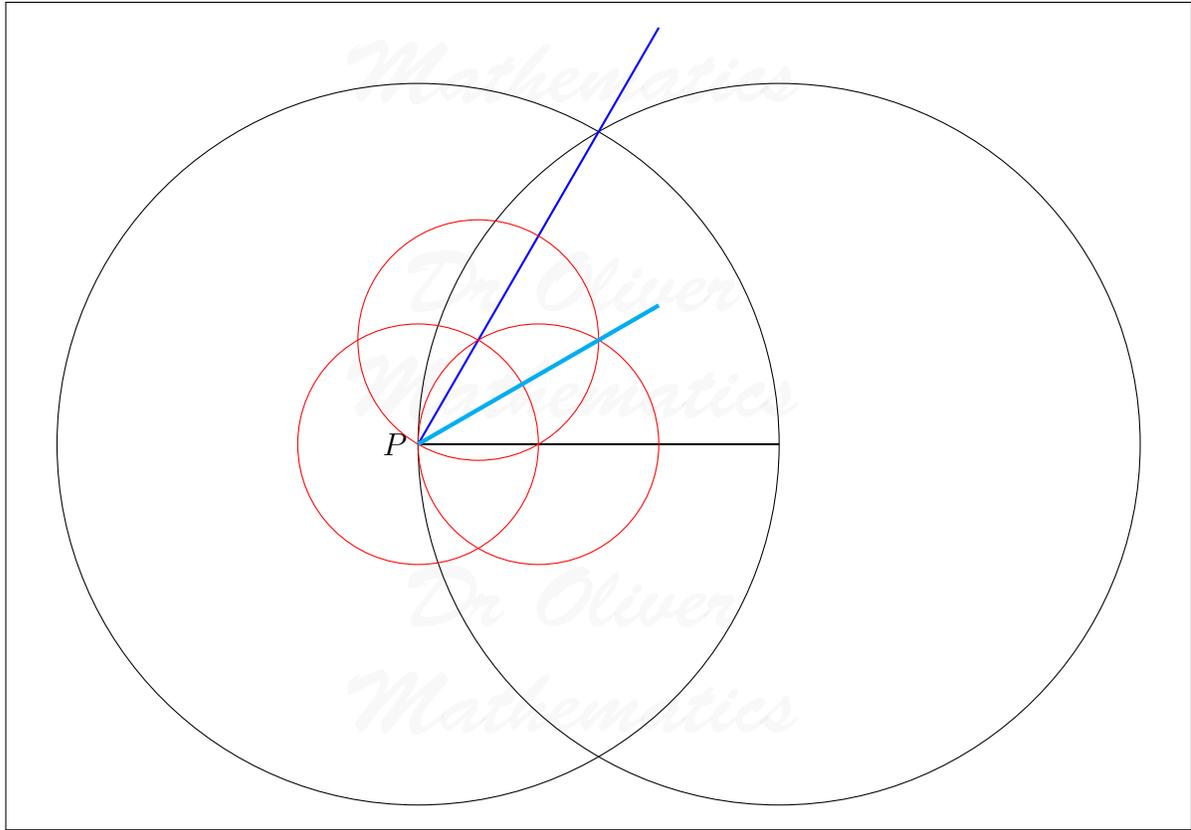Frequency

Weight of box (w kg)

Solution

Frequency

Weight of box (w kg)

14. Use ruler and compasses to **construct** an angle of $30°$ at P. (3)
    You **must** show all your construction lines.

P ——————————————

Solution

15. (a) Expand (2)
$$x(x + 2).$$

Solution
$$x(x + 2) = \underline{\underline{x^2 + 2x}}.$$

(b) Expand and simplify (2)
$$(x + 3)(x - 4).$$

Solution

$\times$	x	$+3$
x	x^2	$+3x$
-4	$-4x$	-12

$$(x+3)(x-4) = \underline{\underline{x^2 - x - 12}}.$$

(c) Factorise completely   (2)

$$2y^2 - 4y.$$

Solution

$$2y^2 - 4y = \underline{\underline{2y(y-2)}}.$$

(d) Factorise   (1)

$$x^2 - 9.$$

Solution

$$x^2 - 9 = x^2 - 3^2 = \underline{\underline{(x+3)(x-3)}}.$$

16. (a) Work out   (3)

$$\tfrac{2}{3} \div \tfrac{5}{6}.$$

Give your fraction in its simplest form.

Solution

$$\tfrac{2}{3} \div \tfrac{5}{6} = \tfrac{2}{3} \times \tfrac{6}{5}$$
$$= \tfrac{12}{15}$$
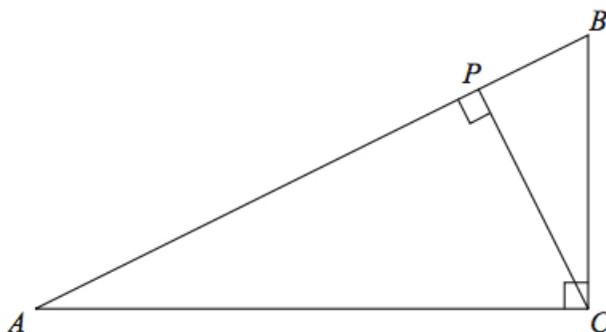$$= \underline{\underline{\tfrac{4}{5}}}.$$

(b) Work out   (3)

$$2\tfrac{1}{3} - 1\tfrac{2}{5}.$$

Solution

14

$$2\tfrac{1}{3} - 1\tfrac{2}{5} = 1 + \tfrac{5}{15} - \tfrac{6}{15}$$
$$= 1 - \tfrac{1}{15}$$
$$= \underline{\underline{\tfrac{14}{15}}}.$$

17. In the diagram, ABC is a triangle, angle $ACB = 90°$, P lies on the line AB, and CP is   (3)
perpendicular to AB.

Prove that the angles of triangle APC are the same as the angles of triangle CPB.

Solution

$\angle APC = \angle CPB = 90°$.
$\angle PCB = 90° - \angle PCB = \angle PAC$.
$\angle PBA = 90° - \angle PAC = \angle PCA$.
Hence, the angles of triangle APC are the same as the angles of triangle CPB.

18. The table shows information about the time, m minutes, it takes to show each of 120 films.

Time (m minutes)	Frequency
$70 < m \leqslant 80$	4
$80 < m \leqslant 90$	12
$90 < m \leqslant 100$	34
$100 < m \leqslant 110$	32
$110 < m \leqslant 120$	26
$120 < m \leqslant 130$	12

15

(a) Write down the modal class interval. (1)

(b) Complete the cumulative frequency table. (1)

Time (m minutes)	Cumulative Frequency
$70 < m \leqslant 80$	4
$70 < m \leqslant 90$	
$70 < m \leqslant 110$	
$70 < m \leqslant 120$	
$70 < m \leqslant 130$	

(c) On the grid, draw a cumulative frequency graph for your cumulative frequency (2)
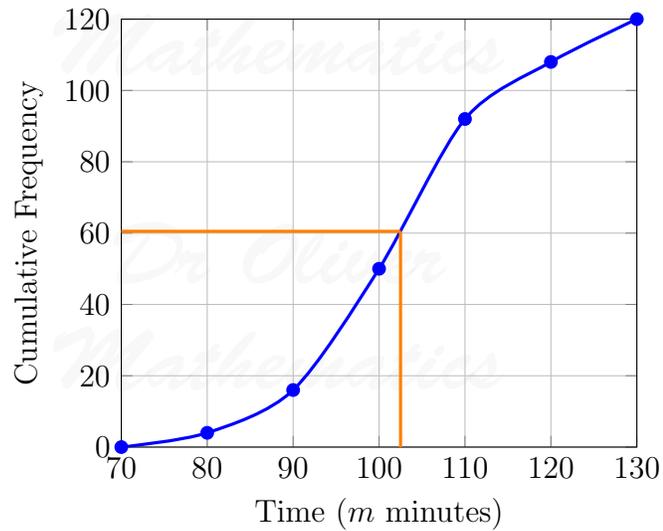table.
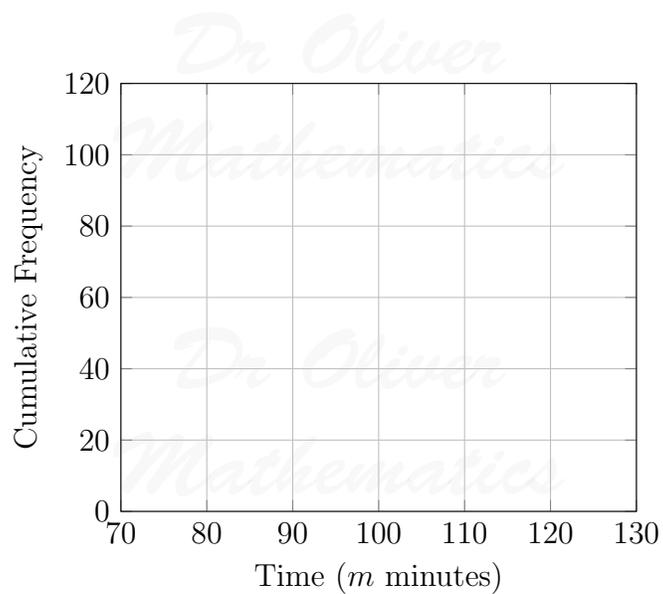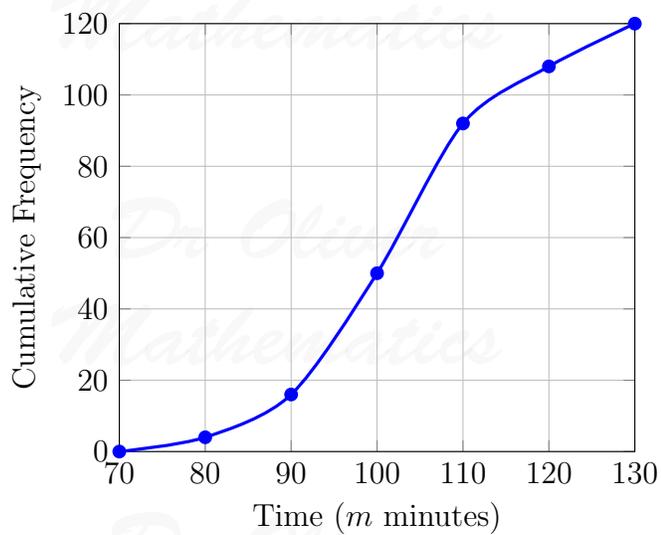
Solution

(d) Use your graph to find an estimate for the median. (1)

Solution

Draw from

$$\frac{120 + 1}{2} = 60.5 :$$

Correct read-off: approximately <u>102.5 minutes</u>.

19. Solve the simultaneous equations (3)

$$4x + y = 10$$
$$2x - 3y = 19.$$

Solution

$$4x + y = 10 \quad (1)$$
$$2x - 3y = 19 \quad (2)$$

E.g., $(1) - 2 \times (2)$:
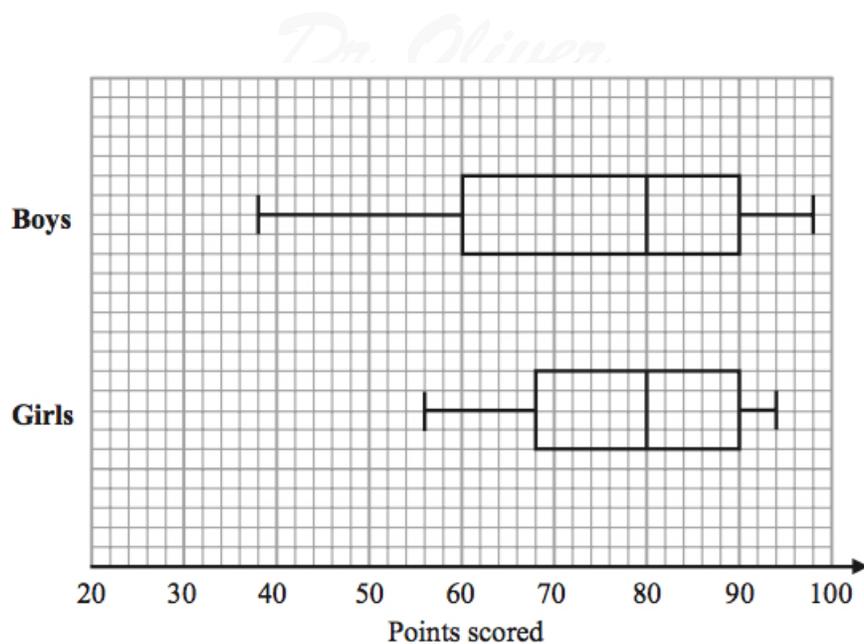
$$7y = -28 \Rightarrow \underline{\underline{y = -4}}$$
$$\Rightarrow 4x - 4 = 10$$
$$\Rightarrow 4x = 14$$
$$\Rightarrow \underline{\underline{x = 3\tfrac{1}{2}}}.$$

20. The box plots show information about the points scored by some students in a spelling competition. (2)

Compare the distributions of the boys' scores and the girls' scores.

Solution

Median: As the boys' median is the same as the girls' median (80), they did equally well in the spelling competition.

IQR: As the girls' IQR ($90 - 68 = 22$) is smaller than the boys' ($90 - 60 = 30$), the scores were more consistent for the girls.
OR
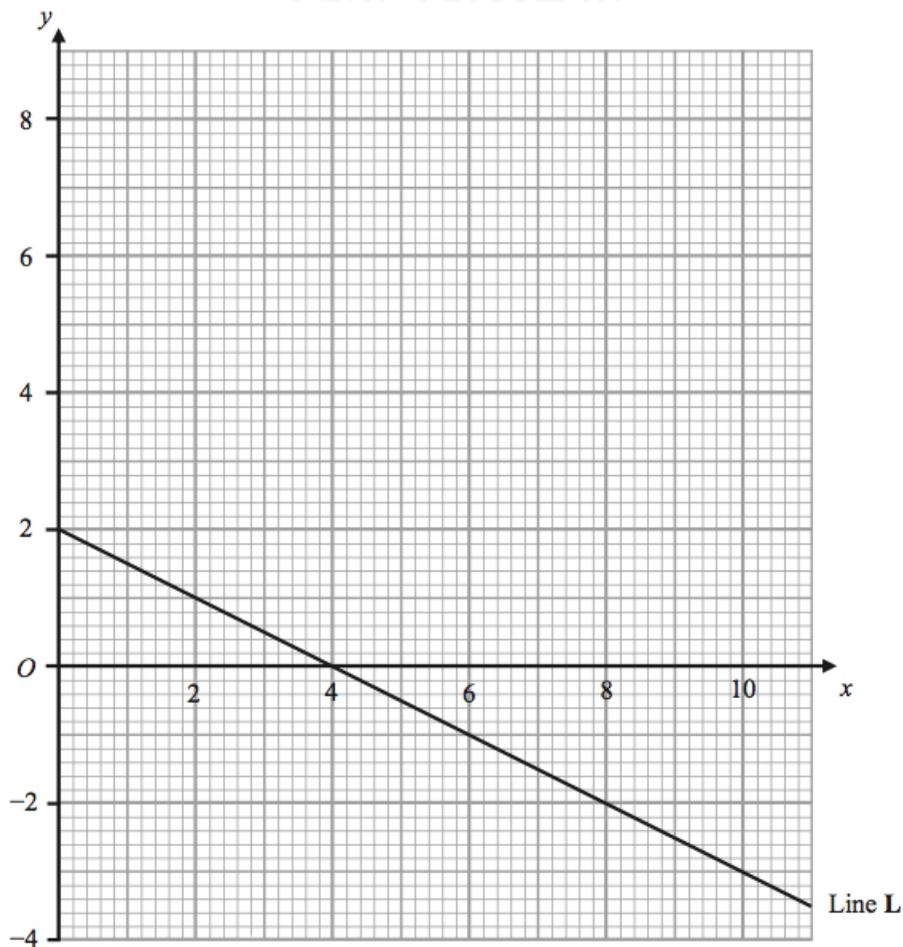**Range**: As the girls' range ($94 - 56 = 38$) is smaller than the boys' ($98 - 38 = 60$), the scores were more consistent for the girls.

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21. Line **L** is drawn on the grid.

(a) Work out the gradient of Line **L**. (2)

Solution

$$\text{Gradient} = \frac{2 - (2)}{0 - 8}$$
$$= \frac{4}{-8}$$
$$= -\frac{1}{2}.$$

Another line, Line **M**, is parallel to Line **L** and passes through the point $(6, 2)$.

(b) Find an equation for Line **M**. (2)

Solution

$$y - 2 = -\tfrac{1}{2}(x - 6) \Rightarrow y - 2 = -\tfrac{1}{2}x + 3$$
$$\Rightarrow \underline{\underline{y = -\tfrac{1}{2}x + 5.}}$$

22. (a) Find the value of $27^{-\frac{2}{3}}$. (2)

Solution

$$27^{-\frac{2}{3}} = \frac{1}{(\sqrt[3]{27})^2}$$
$$= \frac{1}{3^2}$$
$$= \underline{\underline{\tfrac{1}{9}}}.$$

(b) Given that (3)
$$\frac{8 - \sqrt{18}}{\sqrt{2}} = a + b\sqrt{2},$$

where a and b are integers, find the value of a and the value of b.

Solution

$$\frac{8 - \sqrt{18}}{\sqrt{2}} = \frac{8 - \sqrt{9 \times 2}}{\sqrt{2}}$$
$$= \frac{8 - 3\sqrt{2}}{\sqrt{2}}$$
$$= \frac{8 - 3\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{8\sqrt{2} - 6}{2}$$
$$= \underline{\underline{4\sqrt{2} - 3}};$$

hence, $\underline{\underline{a = -3}}$ and $\underline{\underline{b = 4}}$.

23. Make k the subject of the formula (4)

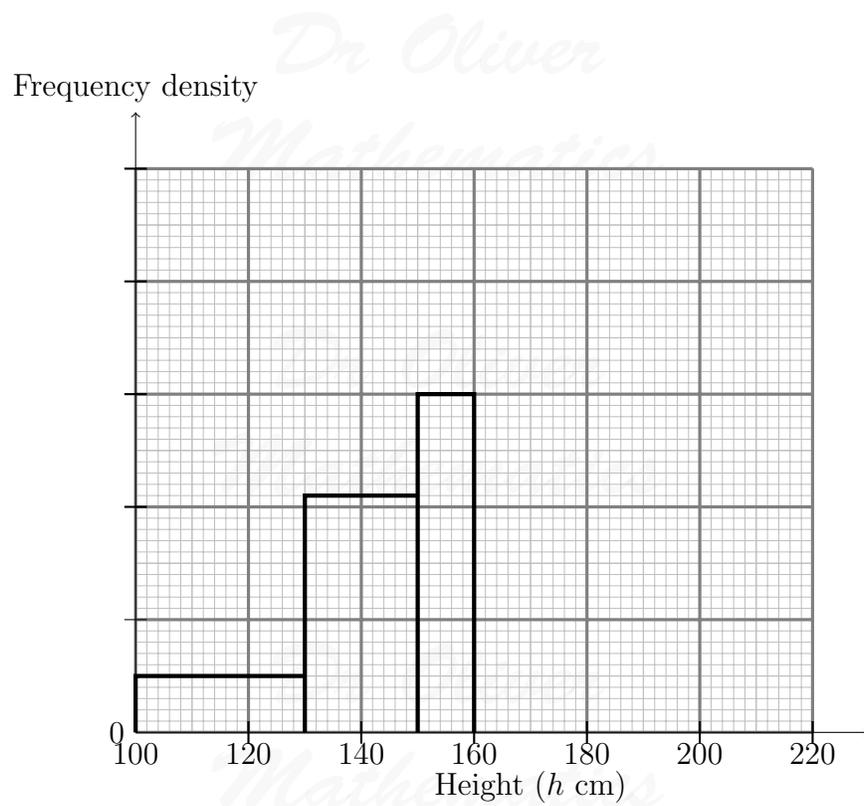$$t = \frac{k}{k-2}.$$

24. The incomplete table and histogram give some information about the heights (in cm) of some sunflowers.

Height (h cm)	Frequency
$100 < m \leqslant 130$	30
$130 < m \leqslant 150$	
$150 < m \leqslant 160$	
$160 < m \leqslant 180$	40
$180 < m \leqslant 210$	18

22

Frequency density

Height (h cm)

(a) Use the histogram to complete the table. (2)

Solution

Height (h cm)	Frequency	Width	Frequency Density
$100 < m \leqslant 130$	30	30	$\frac{30}{30} = 1$
$130 < m \leqslant 150$		20	
$150 < m \leqslant 160$		10	
$160 < m \leqslant 180$	40	20	$\frac{40}{20} = 2$
$180 < m \leqslant 210$	18	30	$\frac{18}{30} = 0.6$

Frequency density
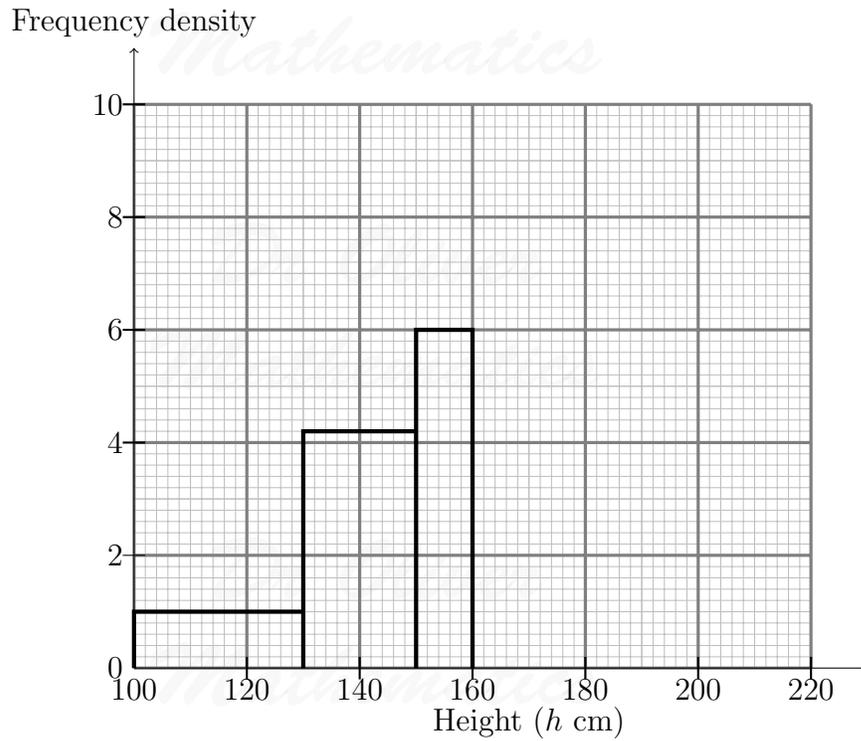
Height (*h* cm)

Height (*h* cm)	Frequency	Width	Frequency Density
$100 < m \leqslant 130$	30	30	$\frac{30}{30} = 1$
$130 < m \leqslant 150$	84	20	$\frac{84}{20} = 4.2$
$150 < m \leqslant 160$	60	10	$\frac{60}{10} = 6$
$160 < m \leqslant 180$	40	20	$\frac{40}{20} = 2$
$180 < m \leqslant 210$	18	30	$\frac{18}{30} = 0.6$

(b) Use the table to complete the histogram. (2)
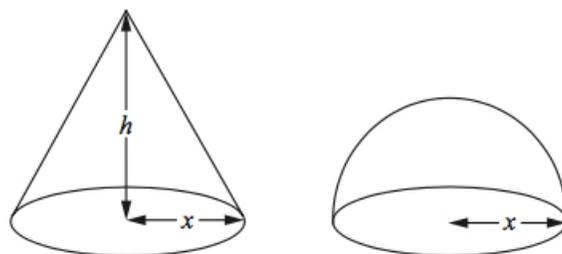
Solution

Frequency density

Height (h cm)
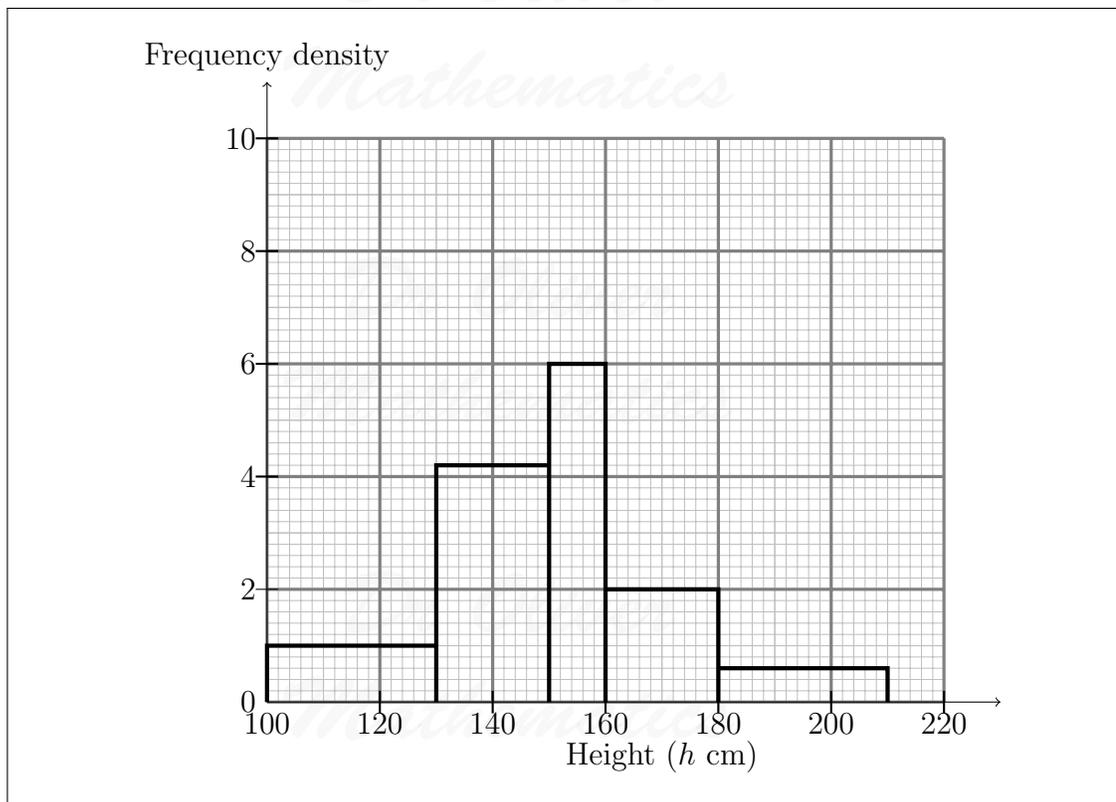
25. The diagram shows a solid cone and a solid hemisphere. (4)

The cone has a base of radius x cm and a height of h cm.
The hemisphere has a base of radius x cm.
The surface area of the cone is equal to the surface area of the hemisphere.
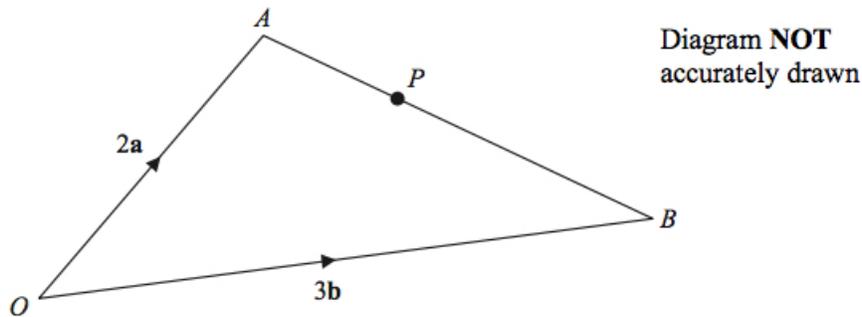Find an expression for h in terms of x.

Solution

Cone: Surface area is $\pi l x + \pi x^2$ (where l is the slant height).
Hemisphere: Surface area is $2\pi x^2 + \pi x^2$.

Now,

$$\pi l x + \pi x^2 = 2\pi x^2 + \pi x^2 \Rightarrow \pi l x = 2\pi x^2$$
$$\Rightarrow l = 2x$$
$$\Rightarrow l^2 = 4x^2$$
$$\Rightarrow h^2 + x^2 = 4x^2$$
$$\Rightarrow h^2 = 3x^2$$
$$\Rightarrow \underline{\underline{h = \sqrt{3}x}}.$$

26. OAB is a triangle.

Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 2\mathbf{a}$.
$\overrightarrow{OB} = 3\mathbf{b}$.

(a) Find $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (1)

Solution

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$= \underline{\underline{-2\mathbf{a} + 3\mathbf{b}}}.$$

P is the point on AB such that $AP : PB = 2 : 3$.

(b) Show that $\overrightarrow{OP}$ is parallel to the vector $\mathbf{a} + \mathbf{b}$. (3)

Solution

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$
$$= \overrightarrow{OA} + \tfrac{2}{5}\overrightarrow{AB}$$
$$= 2\mathbf{a} + \tfrac{2}{5}(-2\mathbf{a} + 3\mathbf{b})$$
$$= 2\mathbf{a} - \tfrac{4}{5}\mathbf{a} + \tfrac{6}{5}\mathbf{b}$$
$$= \tfrac{6}{5}\mathbf{a} + \tfrac{6}{5}\mathbf{b}$$
$$= \underline{\underline{\tfrac{6}{5}(\mathbf{a} + \mathbf{b})}},$$

as required.

27. Solve the equation (4)

$$\frac{x}{2} - \frac{2}{x + 1} = 1.$$

Solution

Multiply each side by $2(x + 1)$:

$$\frac{x}{2} - \frac{2}{x + 1} = 1 \Rightarrow x(x + 1) - 4 = 2(x + 1)$$
$$\Rightarrow x^2 + x - 4 = 2x + 2$$
$$\Rightarrow x^2 - x - 6 = 0$$

$$\left.\begin{array}{l} \text{add to:} \quad -1 \\ \text{multiply to:} \quad -6 \end{array}\right\} - 3, +2$$

$$\Rightarrow (x - 3)(x + 2) = 0$$
$$\Rightarrow x - 3 = 0 \text{ or } x + 2 = 0$$
$$\Rightarrow \underline{\underline{x = 3 \text{ or } x = -2}}.$$