Dr Oliver Mathematics
Quadratic Sequences

In this note, we investigate quadratic sequences. The easiest way to do this is to do a few examples.

1. Here are the first five terms of an quadratic sequence:

$$0 \quad 7 \quad 18 \quad 33 \quad 52.$$

Find, in terms of n, an expression for the nth term of this sequence.

Solution

Let the

$$n\text{th term} = an^2 + bn + c.$$

Write down the sequence:	0	7	18	33	52
First line of differences:	7	11	15	19	
Second line of differences:	4	4	4		

Compare this with the first five $(an^2 + bn + c)$s:

$$\begin{aligned}
n = 1: &\quad a \times 1^2 + b \times 1 + c = a + b + c,\\
n = 2: &\quad a \times 2^2 + b \times 2 + c = 4a + 2b + c,\\
n = 3: &\quad a \times 3^2 + b \times 3 + c = 9a + 3b + c,\\
n = 4: &\quad a \times 4^2 + b \times 4 + c = 16a + 4b + c, \text{ and}\\
n = 5: &\quad a \times 5^2 + b \times 5 + c = 25a + 5b + c.
\end{aligned}$$

Table 1 displays the rows, and first and second line of differences.

We compare terms:

$$2a = 4 \Rightarrow a = 2,$$

$$\begin{aligned}
3a + b = 7 &\Rightarrow 3 \times 2 + b = 7\\
&\Rightarrow b = 1,
\end{aligned}$$

and

$$\begin{aligned}
a + b + c = 0 &\Rightarrow 2 + 1 + c = 0\\
&\Rightarrow c = -3;
\end{aligned}$$

hence,

$$n\text{th term} = \underline{\underline{2n^2 + n - 3}}.$$

2. Here are the first five terms of an quadratic sequence:

$$9 \quad 7 \quad -1 \quad -15 \quad -35.$$

Find, in terms of n, an expression for the nth term of this sequence.

Solution

Let the

$$n\text{th term} = an^2 + bn + c.$$

Write down the sequence:	9	7	-1	-15	-35
First line of differences:	-2	-8	-14	-20	
Second line of differences:	-6	-6	-6		

Table 1 displays the rows, and first and second line of differences.

We compare terms:

$$2a = -6 \Rightarrow a = -3,$$

$$\begin{aligned} 3a + b = -2 &\Rightarrow 3 \times (-3) + b = -2 \\ &\Rightarrow b = 7, \end{aligned}$$

and

$$\begin{aligned} a + b + c = 9 &\Rightarrow -3 + 7 + c = 9 \\ &\Rightarrow c = 5; \end{aligned}$$

hence,

$$n\text{th term} = \underline{\underline{-3n^2 + 7n + 5}}.$$

3. Here are the first five terms of an quadratic sequence:

$$x \quad 16 \quad y \quad 80 \quad 127.$$

Find x and y.

Solution

Let the

$$n\text{th term} = an^2 + bn + c.$$

x	16	y	80	127
$16-x$	$y-16$	$80-y$	47	
$x+y-32$	$96-2y$	$y-33$		

We compare terms:

$$96-2y = y-33 \Rightarrow 129 = 3y$$
$$\Rightarrow \underline{\underline{y = 43}};$$

now,

$$96-2y = 96-2\times 43 = 10,$$

and

$$x+y-32 = 10 \Rightarrow x+43-32 = 10$$
$$\Rightarrow \underline{\underline{x = -1}}.$$

Write down the sequence:	$a+b+c$	$4a+2b+c$	$9a+3b+c$	$16a+4b+c$	$25a+5b+c$
First line of differences:	3a+b	$5a+b$	$7a+b$	$9a+b$	
Second line of differences:	2a	$2a$	$2a$		

Table 1: the first five entries of an^2+bn+c